

# Joint Adaptation of Frequency Hopping and Transmission Rate for Anti-jamming Wireless Systems

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**Abstract**—Wireless transmissions are inherently vulnerable to jamming attacks. Frequency hopping (FH) and transmission rate adaptation (RA) have been separately used to mitigate jamming. When RA is used alone, it has been shown that a jammer who randomizes its power levels can force the transmitter to *always* operate at the lowest rate, by maintaining the average jamming power above a certain threshold. On the other hand, when only FH is used, a high throughput overhead is incurred due to frequent channel switching. In this paper, we propose to mitigate jamming by jointly optimizing the FH and RA techniques. This way, the transmitter can escape the jammer by changing its channel, adjusting its rate, or both. We consider a power-constrained “reactive-sweep” jammer who aims at degrading the throughput of the wireless link. The jammer sweeps through the set of channels, jamming a subset of them at a time, using the optimal jamming power. We model the interactions between the legitimate transmitter and jammer as a constrained zero-sum Markov game. The transmitter’s optimal defense strategy is derived by obtaining the equilibria of the constrained Markov game. This policy informs the transmitter when to hop to another channel and when to stay on the current channel. Furthermore, it gives the best transmission rate to use in both cases (hop or stay). The structure of the transmitter’s optimal policy is shown to be threshold type, whereby the transmitter stays on the same channel up to a certain number of time slots after which it hops. We analyze the “constrained Nash equilibrium” of the Markov game and show that the equilibrium defense strategy of the transmitter is deterministic. Numerical investigations show that the new scheme improves the average throughput and provides better jamming resiliency.

**Index Terms**—Dynamic frequency hopping, jamming, Markov decision processes, Markov games, rate adaptation.



## 1 INTRODUCTION

The broadcast nature of wireless communications leaves them vulnerable to various security threats, including jamming attacks. Adversaries can use readily available off-the-shelf commercial products to launch stealth jamming attacks [1]–[3]. In such attacks, an adversary injects interfering power into the wireless medium, hindering legitimate transmissions in one of two ways: (i) the jamming power can degrade the signal-to-interference-plus-noise ratio (SINR) at a legitimate receiver, or (ii) in carrier-sensing networks, continuous jamming may prevent the legitimate transmitter from accessing the medium, creating a DOS attack. In this paper, we are concerned with the first type of attacks.

Several jamming behaviors have been studied in the literature, including random, constant, proactive, and reactive jamming (see, for example, [2], [4] and the references therein). In this paper, we consider a time-

slotted multi-channel reactive jammer that can listen to various channels and react accordingly. We assume packetized transmissions in which each data packet is explicitly acknowledged by the receiver (many wireless communication systems, such as Wi-Fi, 4G, and LTE are packet-switched). The strategy adopted by the jammer to detect link activity depends on whether the jammer is in the range of the transmitter and/or receiver<sup>1</sup>. If the jammer is in the transmitter’s range, it learns about the link activity by listening to the channels at the beginning of each time slot. On the other hand, if the jammer can listen only to the receiver, then it learns about the link activity by listening to the feedback messages from the receiver. In this paper, we primarily focus on the latter case, which arises in several scenarios, e.g., when the jammer is hidden from the transmitter but not the receiver, or when the transmitter prevents the jammer from listening to it by using beam-forming techniques. In particular, we consider a “reactive-sweep” jammer that attacks the legitimate receiver by jamming several channels in each time slot. Unlike a typical sweep jammer, which continuously sweeps through channels irrespective of the previous jamming outcomes, the considered

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1. The terms ‘transmitter/receiver’ are used to denote the nodes that transmit/receive the data packets.

reactive-sweep jammer changes its sweep strategy based on its listening outcome at the end of each slot. If link activity is detected on a given channel, the jammer switches from a multi-channel attack (where it distributes its power among  $m$  channels) to a single-channel attack during the subsequent slot (where the jammer concentrates its power on one channel). If the transmitter hops away from the jammed channel, the jammer resorts back to its multi-channel jamming strategy. Although transmitting continuously at the maximum power enables the jammer to cause the maximum harm, this happens at the cost of high energy consumption and, more importantly, a high likelihood of being detected. Hence, in this work, we assume a power-constrained jamming model. More details of this model will be given in Section 3.2.

Frequency hopping (FH) [5], [6] and rate adaptation (RA) [7], [8] are commonly used techniques to mitigate jamming. However, these techniques are shown to be ineffective when applied separately [9]–[11]. In the case of RA with no FH, it was shown in [9] that by randomizing its power levels while maintaining the average jamming power above a certain threshold, the jammer can force the transmitter to *always* operate at the lowest rate. Experiments on IEEE 802.11 networks with different RA schemes (e.g., SampleRate [12], AMRR [13], Onoe [14]) also confirm this observation. On the other hand, it was shown in [10], [11] that FH is inadequate in coping with jamming attacks on 802.11 networks. In particular, when the number of channels is small and channels are not perfectly orthogonal, the jammer can degrade the average link throughput significantly [10], [11]. Many wireless systems (e.g., IEEE 802.11) are equipped with both FH and RA capabilities, but these capabilities are not used in an integrated fashion against jamming attacks. Our aim in this paper is to study the effectiveness of a jointly optimized RA and FH technique to mitigate jamming.

One aspect of FH that has been largely overlooked is that it results in a throughput reduction due to channel switching [15] (i.e., “settling time” that is needed when switching between channels). This loss depends on the specific implementation. If the hopping rate is too high, a significant throughput loss will be incurred. On the other hand, operating on the same channel for a longer period increases the risk of being hit by the sweep jammer. In adapting its transmission rate, the transmitter faces a similar dilemma. A high rate increases the jammer’s chances of corrupting the packet. On the other hand, a lower rate increases the signal’s robustness to jamming, but reduces the throughput. We seek to derive a jointly optimal FH and RA policy for the transmitter against a reactive-sweep jammer. This policy informs the transmitter when to switch (hop) to another channel and when to continue (stay) on the current channel. It also determines the best transmission rate to use in

both cases (hop or stay). By optimizing the hop and stay decisions taken by the transmitter, we minimize the hop/switching rate of the transmitter’s channel. Hence, the throughput loss incurred due to channel switching is minimized and the throughput is maximized.

**Main Contributions**—The main contributions of the paper are as follows:

- We model the interactions between a legitimate transmitter and a power-constrained reactive-sweep jammer as a constrained zero-sum Markov game. The transmitter dynamically decides when to switch the operating channel and what transmission rate to use. On the other hand, the jammer dynamically adjusts its jamming power level while satisfying an average jamming power constraint.
- The optimal defense strategy of the transmitter is derived by obtaining the equilibria of the constrained Markov game, and the structure of the optimal policy is shown to be threshold type. Specifically, it is “optimal” for the transmitter to stay on a channel up to a maximum number of time slots after which it needs to hop. We analyze the “constrained Nash equilibrium (NE)” of the Markov game and show that the equilibrium defense strategy of the transmitter is deterministic.
- We compare the average throughput and success rate (percentage of unjammed transmissions) under the proposed technique with a game-theoretic dynamic FH design (with no RA). Through numerical investigations, we show that the new scheme significantly improves the average throughput and jamming resiliency, especially when the number of channels is small and/or the hopping cost is high. For instance, when the number of channels is between 3 and 6: (i) the throughput gain achieved by the new scheme ranges from  $\sim 15x$  to  $\sim 1.37x$ , and (ii) the improvement in the success rate varies from  $\sim 1.7x$  to  $\sim 1.13x$ .

**Paper Organization**—The rest of the paper is organized as follows. We discuss the related work in Section 2. The channel, link, and jamming models are presented in Section 3. In Section 4, we use a zero-sum Markov game to study the interactions between the transmitter and jammer. Using this game, we derive the optimal defense strategies in Section 5. Our numerical experiments are presented in Section 6. Finally, in Section 7 we conclude the paper and provide directions for future research.

## 2 RELATED WORK

In this section, we briefly review existing works on anti-jamming FH and RA techniques. As explained before, current techniques in the literature employ either FH or RA to mitigate jamming, but not both.

**FH-based Anti-jamming Schemes**—Several FH schemes have been proposed in the literature to mitigate jamming. In [5] and [1], two reactive FH schemes were proposed. In these schemes, the legitimate transmitter and receiver hop to a new channel once they determine that the current channel is jammed. A proactive anti-jamming FH scheme was proposed in [6], in which the transmitter and receiver proactively hop between channels without attempting to verify the status of the channels hopped from/to. Compared to a proactive approach, reactive FH minimizes the hopping rate. However, in proactive FH, the legitimate transmitter and receiver do not need to detect the presence of a jammer. The problem of anti-jamming FH without pre-shared keys was studied in [16]. The authors in [16] proposed an uncoordinated FH (UFH) scheme, in which the transmitter and receiver perform random FH. Other UFH schemes were proposed in [17], [18]. The efficiency of UFH-based communication was studied in [19]. In [20], coordinated and uncoordinated FH schemes were proposed for timely delivery of short warning messages in sensor networks. Recently, following a game-theoretic framework, the authors in [21] used FH for jamming mitigation in cognitive radio networks. In [22], the authors developed a combinatorial game-theoretic framework for anti-jamming rendezvous in dynamic spectrum access networks. FH was shown to be largely inadequate in coping with jamming attacks in IEEE 802.11 WLANs [10], [11].

**RA-based Anti-jamming Schemes**—Several algorithms were proposed for RA [23]–[30]. RA for the IEEE 802.11 protocol was investigated in [7], [31], [32]. Experimental and theoretical analysis of optimal jamming strategies against commonly deployed RA schemes in IEEE 802.11 WLANs indicate that the performance can be significantly degraded with very few interfering pulses [9]. To mitigate such interference, the authors in [33] proposed RA and power control schemes. However, the performance of their schemes has not been studied under jamming. The authors in [7] studied the ability of RA and power control in mitigating jamming. Following a game-theoretic framework, it was shown in [9] that if only RA is used to mitigate jamming, then the jammer can force the transmitter to always operate at the lowest rate by merely randomizing its power levels, provided that the average jamming power is above a given threshold. The results in [9] corroborate our claim that RA on a single channel is not effective, and motivates the need for jointly considering RA and FH.

### 3 TRANSMISSION AND JAMMING MODELS

We consider a legitimate transmitter that communicates with its receiver in the presence of a jammer, as shown in Figure 1. The jammer can overhear the receiver’s feedback messages (e.g., ACKs, NACKs), but

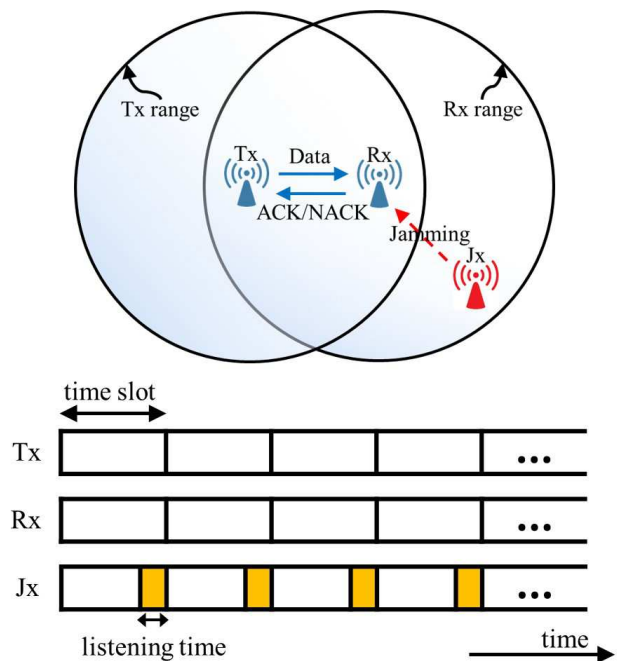


Fig. 1: Transmission model. The jammer (Jx) is within the transmission range of the receiver (Rx) but not the transmitter (Tx). Tx, Rx, and Jx follow a time-slotted FH communication system.

not the transmitter’s messages. This scenario arises in numerous wireless systems, including uplink satellite communications (i.e., ground station to satellite). The footprint of the satellite beam on Earth is typically large. Therefore, a jammer who is close to the ground station can overhear the downlink feedback messages from the satellite. However, because of the directionality of the uplink transmission, the jammer cannot overhear the uplink transmissions from the ground station to the satellite. The scenario in Figure 1 also occurs when the jammer is hidden from the transmitter but not the receiver. This scenario is commonly referred to as ‘hidden terminal problem’ in 802.11 systems [34].

#### 3.1 Transmission Model

We consider a packet-based time-slotted system. During a time slot, the parameters for the transmitter (Tx) and jammer (Jx) are assumed to remain unchanged. The Tx and receiver (Rx) are each equipped with a single radio, and can communicate on any one of  $K$  available channels in a given slot. Let  $\mathcal{F} = \{f_1, f_2, \dots, f_K\}$  denote the set of non-overlapping channels. We assume that the Tx supports  $M + 1$  different transmission rates  $\mathcal{R} = \{R_0, R_1, \dots, R_M\}$ , where  $R_0 < R_1 < \dots < R_M$ . These rates are obtained through different combinations of modulation and coding schemes. In addition to jamming, channel may also experience additive white Gaussian noise with variance  $\sigma^2$ , which is assumed to be the same across all channels. On each channel, the rate that can be

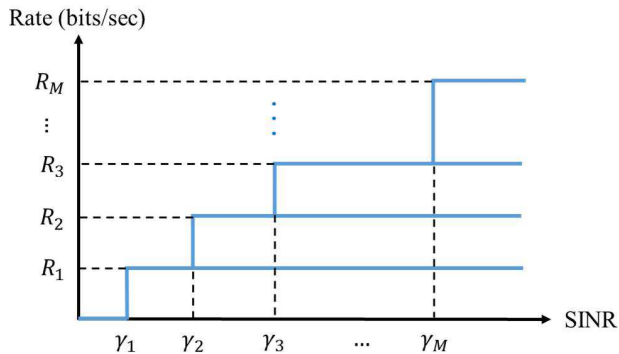


Fig. 2: Rate vs. SINR relationship for data packets.

achieved by the Tx depends on the received SINR. On a given channel, let the received power from the Tx be  $P_R$ , and let  $P_J$  be the jamming power emitted by Jx. The received jamming power at the Rx will be attenuated by a factor  $\alpha$ ,  $0 \leq \alpha \leq 1$ . Then, the SINR at the Rx, denoted by  $\eta$ , is given by:

$$\eta = \frac{P_R}{\alpha P_J + \sigma^2}. \quad (1)$$

Since the main focus in this work is to study the effect of jamming, the above channel model is sufficient to capture the interactions between the Jx and the Tx even though it ignores the effect of channel attenuation. Similar channel model is used in [9], [35] where transmission failures are primarily due to jamming.

For a given received SINR, only certain rates can be decoded at the Rx. The relationship between the “achievable” rates (i.e., rate of a decodable packet) and the SINR is shown in Figure 2. For  $i = 1, \dots, M$ , when  $\gamma_{i-1} \leq \eta < \gamma_i$ , only rates  $R_0, R_1, \dots, R_{i-1}$  can be decoded at the Rx. We assume that the packets are sufficiently long and if the Tx sends a packet at rate  $R_i$  or higher but the received SINR is less than  $\gamma_i$ , the transmitted packet will be completely lost. We also assume the availability of a feedback mechanism from Rx to Tx. If the transmission is successful, the Rx sends an ACK message to the Tx. On the other hand, if transmission fails, the Rx sends a negative ACK (NACK) to the Tx. The ACK/NACK messages can be overheard by the Jx, but we assume they cannot be jammed because of the Tx-Jx distance and the fact that feedback messages are often sent at the lowest transmission rate.

### 3.2 Jamming Model

We consider a time-slotted multi-channel reactive-sweep Jx. The Jx sweeps through the  $K$  channels, jamming  $m$  channels at a time, where  $m \leq K$  (fixed). A typical sweep Jx is depicted in Figure 3 with  $K = 9$  and  $m = 3$  (hence the length of the *sweep cycle* is  $K/m = 3$ ). Consider the first sweep cycle. The Jx attacks channels  $f_1, f_6$ , and  $f_7$  in the first slot,  $f_2, f_4$ , and  $f_9$  in the second slot, and  $f_3, f_5$ , and  $f_8$  in the

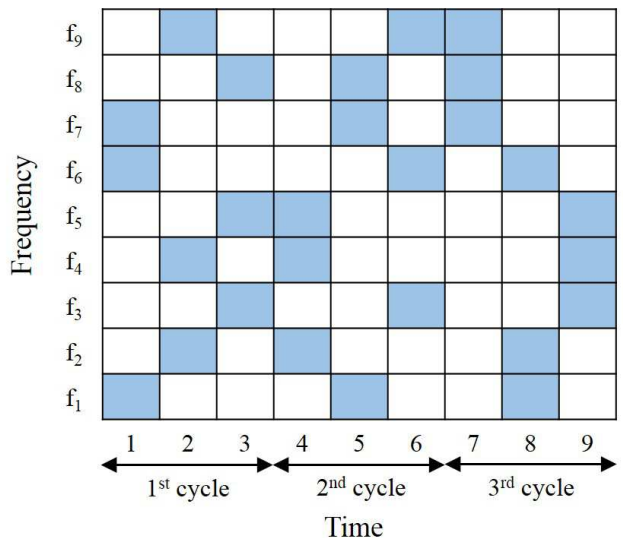


Fig. 3: Example of a typical sweep Jx ( $K = 9$  and  $m = 3$ ). Three sweep cycles are shown.

third slot. Note that in each slot, the Jx selects three channels that have not been jammed since the start of the current sweep cycle. A randomly generated sweep pattern is selected at the start of each new sweep cycle, as can be seen from the second and third sweep cycles in Figure 3.

To implement a reactive-sweep jammer, we modify the classic sweep jamming model as follows. After injecting its interference power into the channel, the Jx listens to ACK/NACK messages over the attacked channels. Detecting an ACK means that the Jx is on the same channel as the Tx-Rx pair but its jamming power was not sufficient to prevent data reception. If the Jx detects a NACK, then the legitimate transmission was successfully jammed. If no messages were detected by the Jx, this means the Tx-Rx have been tuned to a different channel than the Jx’s channel. Based on this outcome, the Jx decides to either: (i) continue sweeping according to the current sweep pattern, (ii) stop sweeping and focus on one channel, or (iii) restart the sweep cycle using a new random sweep pattern. The details of the Jx sweeping strategy will be explained in Section 4.1. When the reactive-sweep Jx decides to stop sweeping, the jamming attack changes from a multi-channel attack (in which the Jx distributes its power among  $m$  channels) to a single-channel attack (in which the Jx emits all of its power on a single channel).

As in [9], we assume a power-limited jamming model (recall that only RA was considered in [9]). In each time slot, Jx can emit on each of the  $m$  channels a maximum power of  $P_{\max}$ . The Jx also has a constraint  $P_{\text{avg}}$  on its time-average power, where  $P_{\text{avg}} < P_{\max}$ . In each time slot, the Jx can choose from  $M + 1$  discrete power levels  $\{P_{J_0}, P_{J_1}, \dots, P_{J_M}\}$ . We assume that the Jx emits the same power level on all  $m$  channels. Let

$\mathcal{M} \stackrel{\text{def}}{=} \{0, 1, \dots, M\}$ .  $P_{J_i}$  is given by:

$$P_{J_i} = \frac{P_R - \sigma^2}{\gamma^{M-i}}, i \in \mathcal{M}. \quad (2)$$

$P_{J_i}, i \in \mathcal{M}$ , is calculated by setting  $\eta$  in (1) to  $\gamma_{M-i}$  in Figure 2 and finding the corresponding  $P_J$  in (1). Similar to [9], under an average-power constraint, the attack strategy is to choose a distribution on the set of available powers that satisfies the average-power constraint. Let  $\mathbf{P}_J$  be the vector that contains the  $J_x$ 's set of pure strategies. Let  $\mathbf{J}_s$  denote the strategy space of the  $J_x$  and  $\mathbb{Y}$  be an  $(M+1)$ -probability simplex. Then,  $\mathbf{J}_s \subset \mathbb{Y}$  and is given by:

$$\mathbf{J}_s = \left\{ \mathbf{y} = (y_0, y_1, \dots, y_M), \sum_{i=0}^M y_i = 1, \mathbf{y} \mathbf{P}_J^T \leq P_{\text{avg}} \right\}. \quad (3)$$

**Switching and Jamming Costs**—When hopping from one channel to another, the Tx needs to remain idle for a brief period, called the *settling time*. The duration of this time period depends on the device (e.g., for the Anthros chipset card, this time is about 7.6 ms [6]). The settling time is required to reconfigure the device on the new channel. Additional loss in throughput occurs due to the lack of synchronization between the Tx and Rx's hopping instances. Collectively, we denote the average loss in throughput due to hopping by  $C$ , and refer to it as *hopping cost*. Outage periods also occur when the Tx is jammed. Jamming disrupts the link between the Tx and the Rx, which needs to be re-established through exchanging several control packets that do not contribute to data throughput. We denote the average loss in throughput due to jamming by  $L$ , and refer to it as *jamming cost*. We account for  $C$  and  $L$  in deriving the optimal defense policy of the Tx.

We assume that the Tx has some prior belief about the  $P_{\text{avg}}$  of the jammer. The Tx used this belief to compute its optimal defense strategy. If the Tx does not have any information about  $P_{\text{avg}}$ , it can set  $P_{\text{avg}} = P_{\text{max}}$ . In this case, the Tx's policy will be pessimistic.

## 4 DYNAMIC FH GAME WITH RA

In this section, we develop a repeated game model between the reactive-sweep  $J_x$  and the Tx. Using this model, we derive the optimal attack (defense) strategy of the  $J_x$  (Tx). We first discuss the attack (defense) strategies that can be adopted by the  $J_x$  (Tx). As noted in [21], these attack and defense strategies mimic an arms race. If the  $J_x$  improves its attack strategy, the Tx counters that with an improved defense strategy, and vice versa. The strategies adopted by each player also depend on hardware and computational resources. Below we discuss the attack and defense strategies, considering the jamming and transmission models of Section 3.

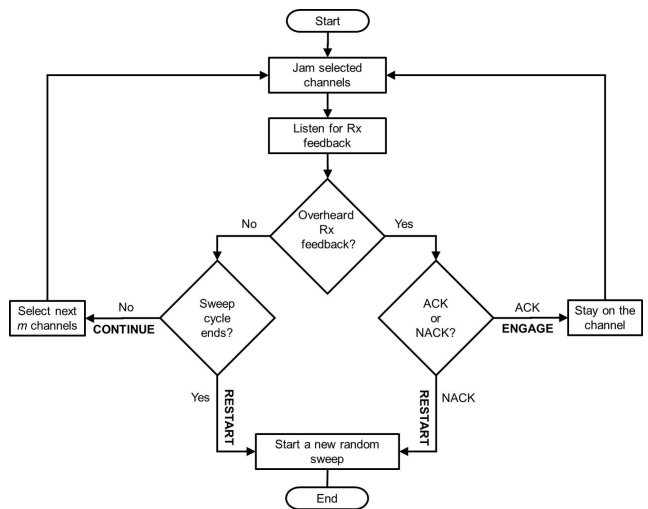


Fig. 4: Illustration of reactive-sweep  $J_x$  in a single slot.

### 4.1 Attack and Defense Strategies

During any time slot, the  $J_x$  emits its jamming power and then listens for an ACK/NACK message at the end of the slot. If the  $J_x$  overhears an ACK/NACK on one of the  $m$  jammed channels, it concludes that the Tx is on that channel<sup>2</sup>. If  $K = 1$ , the only way for the Tx to escape from the  $J_x$  is to adapt its rate. In this case, it is shown in [9] that by randomizing its power levels, the  $J_x$  can enforce the Tx to use the lowest rate. Therefore, when  $K > 1$ , it is better for the Tx to hop to another channel and avoid having to reduce its rate. Knowing this, the  $J_x$  may also decide to hop between channels in search of the Tx.

In [21], a few rounds of arms race have been discussed. It is argued that the best strategy for the  $J_x$  is to sweep through all the  $K$  channels sequentially, jamming  $m$  channels in each slot, and to restart a new sweep cycle with a randomly selected sweep pattern. In our model, we allow the  $J_x$  to further aggravate its attack strategy by making use of its listening capability. Specifically, when  $J_x$  overhears either an ACK or a NACK on a channel, it learns that the Tx is operating on that channel. Accordingly, the  $J_x$  attacks the detected channel, allocating all of its power to that channel until the Tx leaves the channel. Unlike the  $J_x$ , the Tx does not always learn the presence of the  $J_x$  based on the ACK/NACK messages. If a NACK is received, the Tx learns the  $J_x$ 's presence on the channel. However, if an ACK is received, the Tx does not have this information<sup>3</sup>. Therefore, when a NACK is received, it is better for the Tx to hop to a new channel; because otherwise it will be jammed again in the subsequent slot. Being aware of this, the

2. A NACK is generated when the transmission fails either due to a jamming attack or due to a bad channel. We restrict our attention to transmission failures due to jamming attacks.

3. The transmission is successful in the presence of the  $J_x$  if the Tx uses a sufficiently low rate that is de-codable at the given jamming power.

Jx will also leave the channel after receiving a NACK and will start a new sweep cycle with a randomly generated sweep pattern<sup>4</sup>. When the Jx receives an ACK, it continues to stay on the same channel until it either receives a NACK or hears nothing. If the Jx does not hear any feedback messages, it continues with the current sweep cycle. We refer to the Jx that adopts the channel hopping strategy derived in the last round of the aforementioned arms race as a *reactive-sweep Jx*.

We consider a *jamming game* between the Tx and the reactive-sweep Jx. Although the hopping decisions of the Jx are fixed, the Jx still needs to decide about the amount of power to emit in each slot while satisfying its average and maximum power constraints. The Tx's decision consists of what transmission rate to use and also whether to *stay* on the same channel or to *hop* to a new channel. We model the interactions between the Tx and the reactive-sweep Jx as a zero-sum game and derive the optimal attack and defense policies.

## 4.2 Frequency Hopping Strategies

The hopping pattern of the reactive-sweep Jx can be described as follows. The Jx sweeps through the  $K$  channels sequentially, jamming  $m$  non-overlapping channels in each slot. At the end of each slot, the Jx will take one of the following actions based on what it overheard. First, if no ACK/NACK is overheard, Jx continues to jam the next  $m$  channels in the sweep cycle (we refer to this action as 'continue'). If an ACK is overheard on a particular channel, Jx continues to jam only that channel in the next slot, changing its attack from a multi-channel attack to a single-channel attack (we refer to this action as 'engage' since the Jx stops sweeping and keeps attacking the current channel). Finally, if a NACK is overheard or the sweep cycle ends, a new random cycle is restarted immediately (we call this action as 'restart'). The various actions of the reactive-sweep Jx are illustrated in Figure 4.

For the Tx, we assume that it does not have any means to know the quality of various channels and it does not assign priority to any channel. The Tx-Rx pair follows a common FH pattern, generated by a pseudorandom noise (PN) sequence. We note that our optimization of the Tx's channel hopping policy is in terms of how long it stays on a channel (in number of slots) before it hops to a new channel<sup>5</sup>.

## 4.3 Reward

Recall that a data transmission at rate  $R_i$  is successful only if the SINR at the Rx is at least  $\gamma_i$ . If an ACK is received after transmitting at rate  $R_i$ , the Tx obtains a reward of  $R_i$  units. In line with [21], we define the

4. In arriving at this strategy space we assumed a pessimistic scenario where Tx always leaves the jammed channel. However, this pessimistic assumption leads to pessimistic Tx's strategy, and Tx only benefits if the Jx deviates from its strategy.

5. This duration is referred to as the channel residency time in [6].

Tx payoff in a given slot as the difference between the reward and the costs it incurs in that slot. Let  $U_n$  denote the Tx's payoff in slot  $n$ . Then,

$$U_n = R(n) - L \cdot \mathbf{1}[\text{successful jamming}] - C \cdot \mathbf{1}[\text{Tx hops}]$$

where  $R(n) \in \mathcal{R}$  is the transmission rate in slot  $n$  and  $\mathbf{1}[\cdot]$  is the indicator function. An action taken by the Tx in a given slot affects its payoff in future slots. Thus, we will consider a total discounted payoff ( $\bar{U}$ ) with a discount factor  $\delta \in (0, 1)$ , which indicates how much the Tx values its future payoff over its current payoff. Formally,

$$\bar{U} = \sum_n \delta^{n-1} U_n. \quad (4)$$

In the next section, we model the interactions between the Tx and Jx as a zero-sum Markov game and derive the constrained NE (recall that the Jx is average-power constrained). We also characterize the properties of the optimal policies using Markov decision processes (MDPs).

## 5 ZERO-SUM MARKOV GAME

A Markov game is characterized by a state space, an action space, an immediate reward for each player, and transition probabilities. The decision epochs are taken at the end of each time slot, and the effect of the decision takes place at the beginning of the next slot.

### 5.1 Game Formulation

In this section, we formulate our zero-sum Markov game.

#### 5.1.1 State Space

The state of the system identifies the status of the Tx. The Tx's status is described by: (i) determining whether the Tx is jammed or not, and (ii) if not, for how many slots the Tx has been successful on the current channel. Keeping track of the channels that the Tx used in the past and the residency times of these channels is not helpful to the Tx. The reason for this is that while the Tx is operating on a channel, say  $f$ , it does not know which channels the Jx is currently sweeping unless it receives a NACK, and if the Tx is successful on channel  $f$  for  $k$  successive slots, it can only infer that the Jx did not successfully jam channel  $f$  in the last  $k$  slots.

Let  $X$  denote the state space. Then,

$$X = \left\{ J, 1, 2, \dots, \left\lceil \frac{K}{m} \right\rceil \right\} \quad (5)$$

where  $J$  denotes that the Tx is jammed and  $i = 1, 2, \dots, \left\lceil \frac{K}{m} \right\rceil$  denotes that the Tx has been successful on the current channel for the last  $i$  consecutive slots.

### 5.1.2 Action Space

At the end of each slot, the Tx decides whether to stay on the current channel or hop to a new channel. It also decides which rate to use from the set  $\mathcal{R}$ . Therefore, the set of actions available to the Tx for any state in  $X$  is as follows:

$$A = \{(s, R_0), \dots, (s, R_M), (h, R_0), \dots, (h, R_M)\} \quad (6)$$

where  $(s, R_i), i \in \mathcal{M}$  represents the decision to stay on the current channel and use rate  $R_i$ , and  $(h, R_i)$  represents the decision to hop to a new channel and use rate  $R_i$ . For notational convenience, we write  $s_i \stackrel{\text{def}}{=} (s, R_i)$  and  $h_i \stackrel{\text{def}}{=} (h, R_i), \forall i \in \mathcal{M}$ . From the discussion in Subsection 5.2, the Tx takes actions  $\{(h, R_0), \dots, (h, R_M)\}$  once it is in state  $J$ . In all other states, it can take any action in  $A$ .

### 5.1.3 Immediate Reward

$U_n = U_n(x, a_1, a_2, x')$  represents the immediate reward the Tx receives after going from state  $x$  to state  $x'$  when the actions taken by the Tx and the Jx are  $a_1 \in A$  and  $a_2 \in \mathbf{P}_J$ , respectively. This reward does not depend on the slot index, hence we drop the subscript  $n$ . For any  $(a_1, a_2, x) \in A \times \mathbf{P}_J \times X$ , the immediate payoff of the Tx is given by:

$$U(\cdot, a_1, a_2, x') = \begin{cases} -L - C, & \text{if } x' = J, a_1 = h_i, a_2 = P_{J_j}, j > M - i \\ R_i - C, & \text{if } x' = 1, a_1 = h_i, a_2 = P_{J_j}, j \leq M - i \\ -L, & \text{if } x' = J, a_1 = s_i, a_2 = P_{J_j}, j > M - i \\ R_i, & \text{if } x' \neq J, a_1 = s_i, a_2 = P_{J_j}, j \leq M - i. \end{cases}$$

Note that the Tx's reward depends only on the action it takes and the new state it enters, and not on its current state. Since the Jx cannot observe the state of the Tx, it chooses its power level such that its average-power constraint is satisfied.

### 5.1.4 Transition Probabilities

Let  $P(x'|x, a_1, a_2)$  denote the transition probability to state  $x'$  given that the current state is  $x$ , the Tx chooses action  $a_1 \in A$ , and the Jx chooses action  $a_2 \in \mathbf{P}_J$ . First, let us consider the case where the Tx's action involves hopping to a new channel. Let the Tx be on channel  $f$  after hopping. When action  $h_i$  is taken in any state, the state on the new channel can be either  $J$  or  $1$ ,  $\forall i \in \mathcal{M}$ . Let  $x = J$ . Then, on taking action  $h_i$  the system enters state  $J$  again only if the Jx also hops into channel  $f$  and uses a power level that does not allow the Tx to succeed at rate  $R_i$ . Recall that on each successful jamming the Tx hops to a new channel, and the Jx repeats the jamming process with a new sweep pattern that is independent of its past sweep pattern. Then, the Tx and Jx hop to the same channel with probability  $m/(K-1)$ . Hence,  $P(J|J, h_i, P_{J_j}) = 1 - P(1|J, h_i, P_{J_j}), i \in \mathcal{M}$ , is given by:

$$P(J|J, h_i, P_{J_j}) = \begin{cases} m/(K-1), & \text{if } j > M - i \\ 0, & \text{otherwise.} \end{cases} \quad (7)$$

Taking action  $h_i, i \in \mathcal{M}$ , in state  $x \in X \setminus J$ , say  $x = \tilde{x}$ , the Tx's next state can be either  $x' = J$  or  $1$ .  $x'$  will be equal to  $1$  if any of the following happens: (i) channel  $f$  is already swept by the Jx, (ii) channel  $f$  is not swept by the Jx and the Jx does not hop to it in the next time slot, or (iii) the Jx hops to  $f$  and uses a power level that does not disrupt the transmission at rate  $R_i$ . Let  $a_1 = h_i, a_2 = P_{J_j}$  and  $j > M - i$ , then the Tx is successful on  $f$  if the Jx does not hop into  $f$ . Thus, assuming  $j > M - i$

$$\begin{aligned} P(1|x, h_i, P_{J_j}) &= 1 - P(J|x, h_i, P_{J_j}) \\ &= \frac{mx}{K-1} + \frac{K-1-mx}{K-1} \left\{ 1 - \frac{m}{K-1-mx} \right\} \\ &= 1 - m/(K-1). \end{aligned} \quad (8)$$

Therefore,

$$P(1|x, h_i, P_{J_j}) = \begin{cases} 1 - m/(K-1), & \text{if } j > M - i \\ 1, & \text{otherwise.} \end{cases} \quad (9)$$

Now consider the case where the Tx decides to stay on its current channel. Suppose that the Tx is on channel  $f$  and state  $x \in X \setminus J$ , say  $x = \tilde{x}$ , and takes action  $s_i, i \in \mathcal{M}$ . Then, the Tx enters into state  $x' = J$  or  $x' = \tilde{x} + 1$ .  $x'$  will be equal to  $J$  in one of two cases. First, if the Jx did not sweep  $f$  in the last  $\tilde{x}$  slots, hops into  $f$  in the next slot, and jams at a power that does not allow decoding at rate  $R_i$ . Second, if the Jx is already on  $f$  and jams at a power that does not allow decoding at rate  $R_i$ . The probability of the first case can be computed as  $m/(K - m\tilde{x})$  and the probability of the second case is  $m\tilde{x}/K$ . Note that if the jamming power is  $P_{J_j}$  in the first case, then it is  $mP_{J_j}$  in the second case (i.e., single-channel attack).

Let  $\gamma(j, m) \stackrel{\text{def}}{=} \frac{P_R}{\alpha m P_{J_j} + \sigma^2}$ . Then,

$$\begin{aligned} P(J|x, s_i, P_{J_j}) &= 1 - P(x+1|x, s_i, P_{J_j}) \\ &= \begin{cases} \frac{m(x+1)}{K}, & \text{if } x < K/m \text{ and } j > M - i \\ \frac{mx}{K}, & \text{if } x < K/m \text{ and } \gamma(j, m) < \gamma_i \\ 0, & \text{otherwise.} \end{cases} \end{aligned} \quad (10)$$

Next, we introduce the required notations to define the Tx and Jx strategies and their objective functions, following the notational convention of [36]. In each time slot, the Tx takes an action that depends on its past observation. We will only consider Markov stationary policies, where the Tx takes an action based on its current state only (note that the state has information of past history). As stated in [37][Ch. 4], for any given history-dependent policy, there exists a Markov policy that is equally good. The set of Markov stationary policies of the Tx is denoted by  $F_s$ . Let  $\mathcal{M}(A)$  denote the distribution on set  $A$  and  $\mathbf{f} : X \rightarrow \mathcal{M}(A)$  denote the strategy of the Tx. Let  $f(x) \stackrel{\text{def}}{=} \{f(x, a_1), a_1 \in A\}$ , where  $f(x, a_1)$  is the probability of choosing action  $a_1 \in A$  in state  $x \in X$ . Similarly, let the Jx's strategy be  $\mathbf{g} : X \rightarrow \mathbf{P}_J$ , and



let  $g(x) \stackrel{\text{def}}{=} \{g(x, a_2), a_2 \in \mathbf{P}_J\}$ , where  $g(x, a_2)$  is the probability of choosing action  $a_2 \in \mathbf{P}_J$  in state  $x \in X$ . Since the Jx does not know the state, for any Jx's strategy  $\mathbf{y} = (y_0, \dots, y_M) \in J_s$ ,  $g(x) = \mathbf{y}, \forall x \in X$ . Define  $Y_i = \sum_{j>M-i} y_j, i \in \mathcal{M}$ , i.e.,  $Y_i$  denotes the probability that the Jx chooses a power larger than  $P_{J_i}$ .

Let  $r : X \times A \times \mathbf{P}_J \rightarrow \mathbb{R}$  denote the immediate reward for the Tx. For any actions  $a_1$  and  $a_2$  taken by the Tx and Jx, respectively, and any state  $x$ ,  $r(x, a_1, a_2)$  is given by:

$$r(x, a_1, a_2) = \sum_{x'} U(x, a_1, a_2, x') P(x'|x, a_1, a_2). \quad (11)$$

For given  $\mathbf{f} \in F_s$  and  $\mathbf{y} \in J_s$ , the expected discounted payoff of the Tx when the initial state is  $x$  is:

$$\tilde{V}(x, \mathbf{f}, \mathbf{y}) = \mathbb{E}^{\mathbf{f}, \mathbf{y}} \left\{ \sum_n \delta^n r(X_n, A_{1n}, A_{2n}) \mid X_0 = x \right\} \quad (12)$$

where  $\{(X_n, A_{1n}, A_{2n}) : n = 1, 2, \dots\}$  is a sequence of random variables, denoting the state and the actions of the Tx and Jx in each slot, respectively. This sequence evolves according to the policy  $(\mathbf{f}, \mathbf{y})$ . The operator  $\mathbb{E}^{\mathbf{f}, \mathbf{y}}$  denotes the expectation over the process induced by the policies  $\mathbf{f}$  and  $\mathbf{y}$ .

The Tx's objective is to choose a policy  $\mathbf{f}$  that results in the highest expected reward starting from any state  $x \in X$ , and is defined as:

$$V_T(x, \mathbf{y}) = \max_{\mathbf{f} \in F_s} \tilde{V}(x, \mathbf{f}, \mathbf{y}). \quad (13)$$

In contrast, the Jx's objective is to choose a strategy  $\mathbf{y}$  that minimizes the Tx's expected discounted payoff.

$$V_J(x, \mathbf{f}) = \min_{\mathbf{y} \in J_s} \tilde{V}(x, \mathbf{f}, \mathbf{y}). \quad (14)$$

Note that the strategy space of the Jx is constrained, whereas the Tx can choose any stationary policy. A strategy pair  $(\mathbf{f}^*, \mathbf{y}^*)$  is constrained NE if the following two conditions are satisfied:

- $\mathbf{y}^* \in J_s$ ,
- $\forall x \in X, \mathbf{f} \in F_s$ , and  $\mathbf{y} \in J_s$ ,

$$\tilde{V}(x, \mathbf{f}, \mathbf{y}^*) \leq \tilde{V}(x, \mathbf{f}^*, \mathbf{y}^*) \leq \tilde{V}(x, \mathbf{f}^*, \mathbf{y}). \quad (15)$$

Let  $V^*(x) \stackrel{\text{def}}{=} \tilde{V}(x, \mathbf{f}^*, \mathbf{y}^*)$ . Then,  $\{V^*(x), x \in X\}$  is referred to as the value of the zero-sum game<sup>6</sup>.

**Theorem 1:** The zero-sum game has a stationary constrained NE.

*Proof:* While the Jx aims to minimize the Tx's payoff, it needs also to meet its average-power constraint. Since the Jx does not know the value of the current state, its average-power constraint for any strategy  $\mathbf{y}$  (i.e.,  $\mathbf{y} \mathbf{P}_J^T \leq P_{\text{avg}}$ ) can be equivalently written as a

constraint on an expected discounted cost, as follows:

$$C_\beta(\mathbf{f}, \mathbf{y}) = (1 - \beta) \mathbb{E}^{\mathbf{f}, \mathbf{y}} \left\{ \sum_n \beta^{n-1} C(X_n, A_{1n}, A_{2n}) \right\} \leq P_{\text{avg}} \quad (16)$$

for some  $\beta \in (0, 1)$ .  $C(X_n, A_{1n}, A_{2n})$  denotes the cost for the Jx, which is the power it chooses in slot  $n$ , i.e.,  $C(\cdot, \cdot, A_{2n}) = A_{2n}$ . Further, by choosing a strategy  $\mathbf{y}'$  such that  $y'_0 = 1$ , the constraint on the expected discounted cost is strictly met. Thus, strong Slater condition in [38] is verified and the existence of stationary constrained NE follows from Theorem 2.1 in [38].  $\square$

## 5.2 Tx Optimal Defense Strategy

In this section, we study the properties of the Tx's optimal defense strategy against a fixed Jx's strategy. The expected reward of the Tx when the Jx's strategy is  $\mathbf{y}$  is denoted by  $r_{\mathbf{y}} : X \times A \rightarrow \mathbb{R}$ . For a given state-action pair  $(x, a)$ ,  $r_{\mathbf{y}}(x, a)$  is given by:

$$r_{\mathbf{y}}(x, a) = \sum_{i=0}^M y_i r(x, a, P_{J_i}). \quad (17)$$

Let  $P_{\mathbf{y}}(x'|x, a)$  denote the probability that the Tx enters state  $x'$ . Then,

$$P_{\mathbf{y}}(x'|x, a) = \sum_{i=0}^M y_i P(x'|x, a, P_{J_i}). \quad (18)$$

Let  $f_{\mathbf{y}}^*(X)$  denote the policy that maximizes the expected discounted reward function when the Jx uses strategy  $\mathbf{y}$ . Since  $\mathbf{y}$  does not depend on the state, the optimal policy  $f_{\mathbf{y}}^*(X)$  can be obtained by solving a single player MDP with the reward and transition probabilities defined in (17) and (18), respectively. Then,  $f_{\mathbf{y}}^*(X)$  is a deterministic policy [37], i.e.,  $f_{\mathbf{y}}^* : X \rightarrow A$ . For notational convenience, we do not explicitly indicate this dependency on  $\mathbf{y}$ , and write  $V(x) \stackrel{\text{def}}{=} V_T(x, \mathbf{y})$ .

We use the value iteration method [37][Ch. 6] to derive the optimal defense strategy and its properties. The well-known Bellman equations for the expected discounted utility maximization problem in (13) can be written as follows:

$$\begin{aligned} Q(x, a) &\stackrel{\text{def}}{=} r_{\mathbf{y}}(x, a) + \delta \sum_{x' \in X} P_{\mathbf{y}}(x'|x, a) V(x') \\ &= \sum_{x' \in X} P_{\mathbf{y}}(x'|x, a) (r_{\mathbf{y}}(x, a, x') + \delta V(x')) \end{aligned} \quad (19)$$

$$V(x) = \max_{a \in A} Q(x, a).$$

Note that in our formulation, states  $J$  and  $\lceil K/m \rceil$  are equivalent because the Jx will start the sweep cycle afresh and the Tx can only take hop decisions in these states. Hence, when the Tx begins in either state

6. If  $(\tilde{\mathbf{f}}, \tilde{\mathbf{y}})$  is another equilibrium, it also results in the same value of the game [36][Sec. 3.1].



( $J$  or  $\lceil K/m \rceil$ ), it will get the same total discounted reward, i.e.,  $V(J) = V(\lceil K/m \rceil)$ . From (19), for any  $x = J, 1, \dots, K-1$ ,  $V(x)$  is expressed in terms of  $V(J)$  and  $V(x+1)$ . Below we establish the monotonicity of  $V$  on the state space  $X \setminus J$  by restricting the Tx's reward in state  $\lceil K/m \rceil$ , and use this monotonicity property to establish the structure of the optimal policy<sup>7</sup>. For ease of exposition, we provide the analysis for the case  $m = 1$ . Similar analysis follows when  $m > 1$ .

**Lemma 1:**  $V(\cdot)$  is a decreasing function on  $\{1, 2, \dots, K\}$ .

*Proof:* See Appendix A.  $\square$

From (7) and (8), we note that when the Tx takes action  $h_i, i \in \mathcal{M}$ , the probability of entering into state  $J$  or 1 does not depend on the current state. We make use of this observation and the monotonicity of the function  $V(\cdot)$  to derive the following structure of the optimal policy.

**Proposition 1:** The optimal policy  $f^*$  satisfies:

- There exists constants  $K^* \in \{1, \dots, K-1\}$  and  $i^* \leq M$  such that:

$$f^*(x) = h_{i^*} \text{ for } K^* \leq x \leq K-1 \quad \text{and} \quad f^*(0) = s_{i^*}.$$

- For any integers  $x$  and  $y$ , if  $1 \leq x < y < K^*$ ,  $f^*(x) = s_j$ , and  $f^*(y) = s_k$ , then  $j \geq k$ .
- If  $r_{\mathbf{y}}(J, s_i)$  is decreasing in the index  $i$ , then  $i^* = 1$ .

*Proof:* See Appendix B.  $\square$

The above proposition says that when the Tx hops to a new channel it will stay on this channel until it is either jammed or it spent  $K^*$  successive slots on that channel. While operating on a given channel, the Tx adapts its transmission rate—the Tx reduces its rate as the number of successive successful transmissions increases. When the Tx hops, it always uses a fixed rate, which is the maximum available rate ( $R_M$ ) if  $r_{\mathbf{y}}(0, s_i)$  is decreasing in the index  $i$ .

A typical example of the Tx optimal policy is shown in Figure 5 for  $K = 6, M = 2$ , and  $K^* = 4$ . Once jammed, the Tx hops to channel  $f_1$  in the first time slot, and starts communicating over  $f_1$  using the highest rate,  $R_2$ . In the second and third slots, the Tx stays on  $f_1$ , but reduces its rate to  $R_1$ . In the fourth slot, it reduces its rate further to  $R_0$ . After  $K^* = 4$  slots, the Tx hops to channel  $f_3$  and stays on it for only three slots because it gets jammed after that (in slot 8). Then, in the ninth slot, the Tx hops again and tunes to channel  $f_2$  using the highest rate, and so on.

Note that since the Tx hops once it reaches state  $K^*$ , it never enters into a state larger than  $K^*$ . Thus, if  $K^* < K$ , the resulting Markov chain is reducible.

**Corollary 1:** The threshold  $K^*$  is decreasing in  $L$ , and increasing in both  $K$  and  $C$ .

<sup>7</sup> This assumption is made only to establish the structure of the policy analytically. Our simulations show that the same property holds in general.

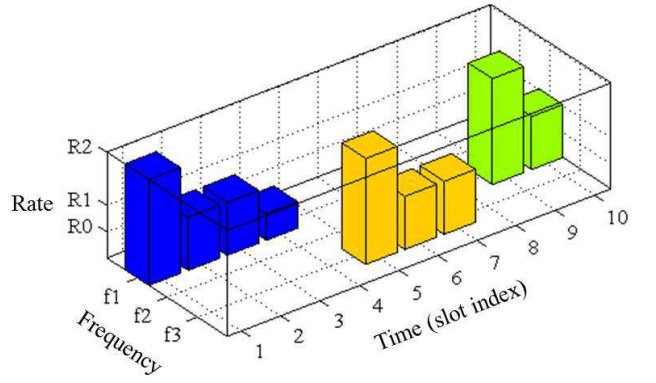


Fig. 5: Example of the Tx optimal policy ( $K = 6, M = 2$ , and  $K^* = 4$ ).

*Proof:* The proof follows by noting that for any  $x' > x$ ,  $Q(x', s_i) - Q(x, s_i)$  is increasing in  $L$  and decreasing in  $K, \forall i \in \mathcal{M}$ . Moreover,  $Q(x, h_i)$  is decreasing in  $C, \forall i \in \mathcal{M}, x \in X$ . This verifies that  $K^*$  is increasing in  $C$ .  $\square$

Next, we return to the study of the Markov game.

### 5.3 Equilibrium of the Markov Game

In this section, we compute the constrained NE of the zero-sum Markov game and study its properties. For a given defense strategy  $\mathbf{y} \in J_s$ , the following linear program solves the recursive equations in (19) [36][Sec 2.3]:

$$\begin{aligned} & \text{minimize} && \frac{1}{N} \sum_x V(x) \\ & \text{subject to:} && V(x) \geq r_{\mathbf{y}}(x, a) + \delta \sum_{x' \in X} P_{\mathbf{y}}(x'|x, a) V(x'), \\ & && \forall x \in X, a \in A. \end{aligned} \quad (20)$$

From Theorem 1, we know that the zero-sum Markov game has a constrained NE. We use a nonlinear version of the above program to compute the equilibria. First, we establish the required notation. Let  $R(x) = [r(x, a, p)]_{a \in A, p \in \mathcal{P}_J}$  and  $T(x, V) = [\sum_{x'} P(x'|x, a, p) V(x')]_{a \in A, p \in \mathcal{P}_J}$  be the reward and transition probability matrices, respectively. Consider the following nonlinear program:

$$\begin{aligned} & \text{minimize} && \frac{1}{N} \sum_x \{V_1(x) + V_2(x)\} \\ & \text{subject to:} && V_1(x) \mathbf{1} \geq R(x) \mathbf{y} + \delta T(x, V_1) \mathbf{y}, \forall x \in X \\ & && V_2(x) \mathbf{1} \geq -f(x) R(x) + \delta f(x) T(x, V_2), \\ & && \forall x \in X \\ & && \mathbf{P}_J \mathbf{y}^T \leq P_{\text{avg}}, \forall x \in X \end{aligned} \quad (21)$$

where  $\mathbf{1}$  denotes a vector of all ones of size  $M$  when the state is  $J$  or  $K$ , and of size  $2M$  for all other states.

**Theorem 2:** Let  $(V_1^*(x), V_2^*(x), f^*(x), \mathbf{y}^*)$  denote the minimum of the nonlinear program (21). Then,

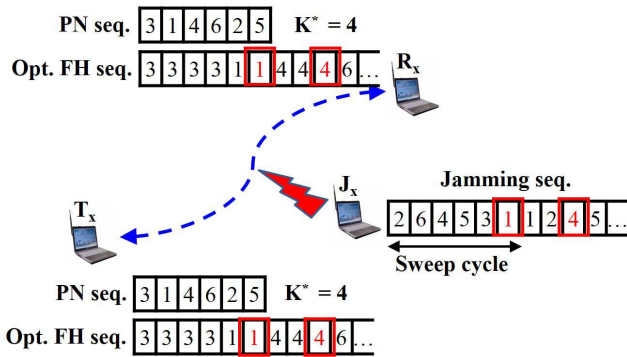


Fig. 6: Example illustrating the Tx-Rx rendezvous process.

$(f^*(x), y^*)$  is the optimal constrained NE of the game.

*Proof:* The nonlinear program (21) is the same as the one in [36][Sec. 3.7] with the additional average-power constraint on the  $J_x$ 's strategy. The proof follows from [36][Th. 3.7.2].  $\square$

Note that although the optimal strategy of the Tx for a given  $J_x$ 's strategy is deterministic, the equilibrium strategy may not be deterministic. The strategy  $y^*$  is the same for all  $x \in X$  as the  $J_x$  does not know the state. We know from the previous subsection that the optimal Tx's strategy against any given  $y$  is deterministic. Therefore, at equilibrium, the strategy of the Tx is deterministic.

**Tx-Rx Rendezvous**—As explained in the paper, the Tx dynamically adjusts its FH sequence according to its optimal anti-jamming strategy. In this case, how does the Rx know the Tx's channel in a given slot?

Initially, the Tx and Rx share a common PN sequence. The Tx follows this PN sequence. However, it optimizes how many slots to stay in each channel before switching to the next channel in the PN sequence. As stated in Proposition 1, the optimal policy of the Tx is to stay on each channel  $K^*$  slots as long as it is not jammed. Once jammed, the Tx will switch to a new channel. The Tx optimal policy is also shared with the Rx. Therefore, the Rx will follow the common PN sequence staying on each channel  $K^*$  slots unless it is jammed, in this case it will switch to the next channel. By knowing the initial PN sequence and  $K^*$ , the Rx can induce the Tx's channel in any time slot, based on its jammed slots. The Tx conveys the value of  $K^*$  to the Rx along with the initial PN sequence before start hopping from one channel to another. The Rx can also obtain the value of  $K^*$  by solving the zero-sum Markov game between the Tx and  $J_x$ . Figure 6 shows an example illustrating the Tx-Rx channel synchronization (rendezvous) process.

## 6 PERFORMANCE EVALUATION

In this section, we study the performance of the joint FH and RA scheme for various system parameters,

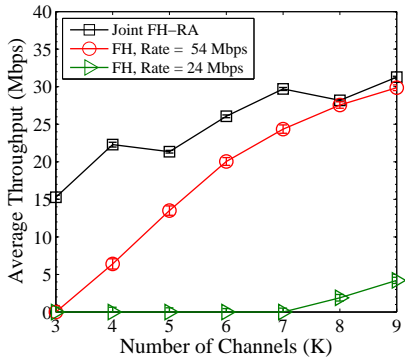
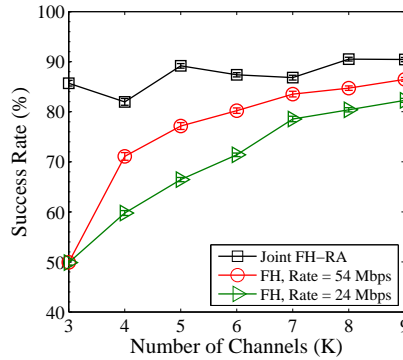
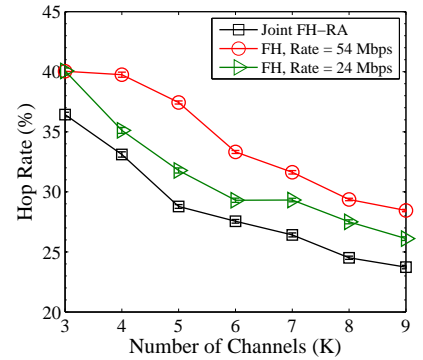
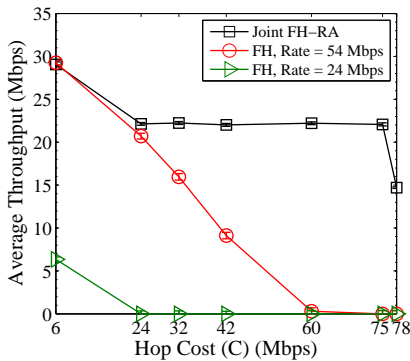
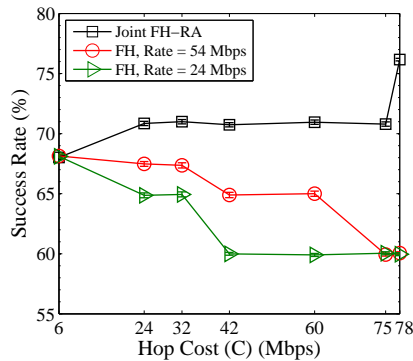
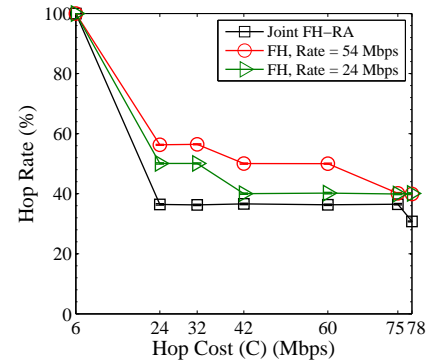
$K$ ,  $C$ ,  $L$ , and  $P_{\text{avg}}$ . We use the set of rates adopted by IEEE 802.11a [39], i.e., 6, 9, 12, 18, 24, 36, 48, and 54 Mbps. Unless stated otherwise, we use the following default parameter values:  $K = 4$ ,  $L = 25$  Mbps,  $C = 50$  Mbps,  $m = 1$ , and  $P_{\text{avg}}/P_{\text{max}} = 25/30$ . The jamming and switching costs are interpreted as throughput loss. Because of this, we represent  $L$  and  $C$  in Mbps. We implement our game in MATLAB. The optimal defense and attack strategies of the Tx and  $J_x$ , respectively, are obtained by solving (21). The 95% confidence intervals are shown in all the simulation figures.

We compare the joint FH and RA scheme with the FH scheme proposed in [21]. In [21], the Tx optimally decides to hop or stay while fixing its transmission rate. We implement two versions of the FH scheme in [21]: one with a fixed transmission rate of 24 Mbps and the other with a fixed rate of 54 Mbps. The FH game in [21] is implemented in MATLAB. The optimal defense and attack strategies of the Tx and  $J_x$ , respectively, are obtained by solving a linear program that is similar to (21). The difference is that the action space of the FH game consists of two actions only: hop and stay, whereas the action space of the joint FH and RA scheme consists of  $2N$  actions, as explained earlier. The joint FH and RA scheme is compared with the FH schemes based on the average throughput (in Mbps), success rate, and the hop rate. Recall that in the RA scheme, the  $J_x$  can force the Tx to always operate at the lowest rate by maintaining the average jamming power above a certain threshold.

### 6.1 Effect of the Number of Channels ( $K$ )

Figure 7 depicts the average throughput vs.  $K$ . The joint scheme achieves a significant improvement in the average throughput compared to the FH schemes. This improvement reaches  $\sim 15x$  when  $K = 3$ . The improvement in the average throughput is due to two factors. First, the increase in the success rate, and accordingly the reduction in the jamming cost, as shown in Figure 8. Second, a reduction in the hop rate, hence a reduction in the switching cost, as shown in Figure 9.

The higher success rate results from the fact that in the joint scheme the Tx can evade the  $J_x$  by hopping to a different channel and/or reducing its transmission rate. Note from Figure 8 that the improvement in the success rate is more significant when  $K$  is small, and it decreases with  $K$ . As a result, the improvement in the average throughput achieved by the joint scheme reduces as  $K$  increases. The reason for this is that RA plays a role in jamming avoidance only when both the Tx and  $J_x$  are on the same channel. As  $K$  increases, the likelihood that the two will select the same channel decreases, hence the gain obtained by the added RA capability is lower. As  $K$  increases, the success rates of the FH schemes improve significantly, because of

Fig. 7: Average throughput vs.  $K$ .Fig. 8: Success rate vs.  $K$ .Fig. 9: Hop rate vs.  $K$ .Fig. 10: Average throughput vs.  $C$ .Fig. 11: Success rate vs.  $C$ .Fig. 12: Hop rate vs.  $C$ .

the reduction in the probability that the Tx and Jx are on the same channel. In contrast, the difference in the success rate of the joint scheme when  $K$  varies from 3 to 9 is not significant. The reason is that even though the probability that the Tx and Jx are on a common channel increases as  $K$  decreases, the Tx uses its RA capability to evade the Jx and keep its success rate high.

Expectedly, the hop rate of the joint scheme is smaller than that of the FH schemes. In the joint scheme, in order to save the hop cost, the Tx tries to evade the Jx by employing RA only (if it can), hence reducing the hop rate as much as it can. The hop rates of all schemes reduce as  $K$  increases since the likelihood of the Tx and Jx meet on a common channel decreases. The reduction in the hop rate of the joint scheme as  $K$  increases causes the average throughput to increase (recall that the success rate of the joint scheme does not improve significantly with  $K$ ).

## 6.2 Effect of the Hop Cost ( $C$ )

The effect of  $C$  on the average throughput, the success rate, and the hop rate is studied in Figures 10, 11, and 12, respectively. The average throughput of the joint scheme (FH schemes) is a nonincreasing (decreasing) function in  $C$ . We vary  $C$  between 6 Mbps (which is the lowest transmission rate) and 78 Mbps

(which is close to 1.5 times the maximum transmission rate). The improvement in the average throughput achieved by the joint scheme reaches to  $\sim 22\times$  when  $C = 75$  Mbps. As can be seen from Figure 10, the improvement in the average throughput achieved by the joint scheme is more significant when  $C$  is sufficiently large. Figure 12 shows that when  $C$  increases, the hop rate of all the schemes decreases. As a result, the success rate of the FH schemes decreases (see Figure 11). In contrast, when the hop rate of the joint scheme decreases (due to the increase in  $C$ ), the success rate remains the same or increases, as shown in Figure 11. This is because of the added RA capability in the joint scheme. When  $C$  is significantly large ( $> 75$  Mbps in Figures 10-12), the Tx tends to avoid jamming by reducing its transmission rate more often than switching to another channel, hence the hop rate reduces, the average throughput decreases, but the success rate increases.

When  $C = 6$  Mbps which is significantly smaller than  $L = 25$  Mbps, the hop rate of all the schemes is 100% and the maximum average throughput is attained.

## 6.3 Effect of the Jamming Cost ( $L$ )

Figures 13, 14, and 15 depict the average throughput, the success rate, and the hop rate of all the schemes vs.  $L$  when  $C = 50$  Mbps. The average throughput

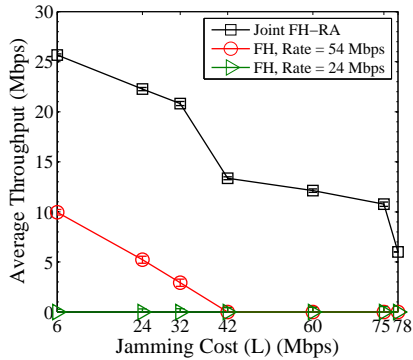


Fig. 13: Average throughput vs.  $L$  ( $C = 50$  Mbps).

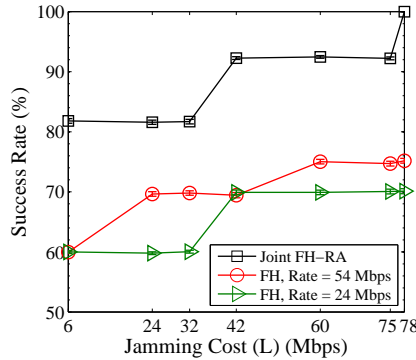


Fig. 14: Success rate vs.  $L$  ( $C = 50$  Mbps).

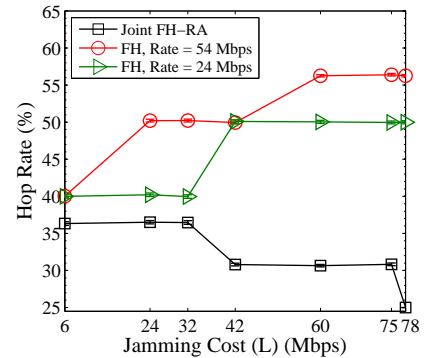


Fig. 15: Hop rate vs.  $L$  ( $C = 50$  Mbps).

of all the schemes decreases with  $L$ . Indeed, the FH scheme with a rate fixed at 24 Mbps results in zero throughput. When  $L = 6$  Mbps which is significantly smaller than  $C = 50$  Mbps, the hop rate of the two FH schemes is  $\sim 40\%$ . When  $L$  increases, the hop rate of the FH schemes increases, and as a result the success rate of the FH schemes increases (see Figures 14 and 15). Although the success rate of the FH schemes improves with  $L$ , the average throughput decreases due to the increase in the hopping cost. In contrast to the FH schemes, when  $L$  increases the hop rate of the joint scheme decreases. This is due to the large value of  $C$ , which leads the Tx to use its RA capability for jamming mitigation instead of hopping (recall from Figure 12 that the hop rate decreases with  $C$ ). As  $L$  increases, the Tx tends to use low rates more often to avoid jamming. As a result, the average throughput of the joint scheme reduces with  $L$ , but the jamming probability also reduces, which causes the hop rate to decrease.

#### 6.4 Effect of the Average Jamming Power ( $P_{\text{avg}}$ )

We study the effect of  $P_{\text{avg}}$  in Figures 16, 17, and 18 with  $P_{\text{max}} = 30$ . For each of the two FH schemes, there is a threshold on  $P_{\text{avg}}$  that depends on the scheme. When  $P_{\text{avg}}$  is less than this threshold, the success rate decreases and the hop rate increases with  $P_{\text{avg}}$ . When  $P_{\text{avg}}$  exceeds this threshold, the success and hop rates become almost unaffected by  $P_{\text{avg}}$ . The reason is that for each fixed transmission rate there is a corresponding average jamming power that enables the Jx to successfully jam the Tx when they are on the same channel. Increasing the average jamming power beyond this threshold does not change the performance of the FH scheme. A similar behavior is also observed for the joint scheme.

Note that when  $P_{\text{avg}}$  is very small, the Tx can successfully transmit at high rates even when it is on the same channel as the Jx. Therefore, the gain achieved by the added RA capability is less significant compared to the case when  $P_{\text{avg}}$  is large.

#### 6.5 Effect of the Number of Jammed Channels per Slot ( $m$ )

For a given  $K$ , increasing  $m$  decreases the length of the sweep cycle. Increasing  $m$   $a$  times is equivalent to decreasing  $K$   $a$  times and keeping  $m = 1$ , e.g.,  $K = 8$  and  $m = 4$  is equivalent to  $K = 2$  and  $m = 1$ . Therefore, increasing  $m$  has the same impact as decreasing  $K$  for the case when  $m = 1$ .

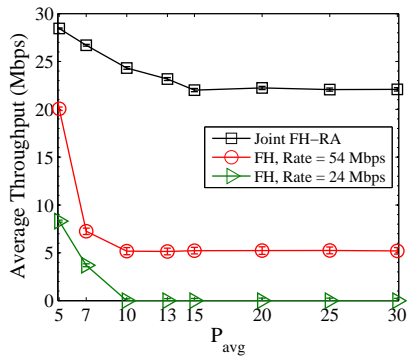
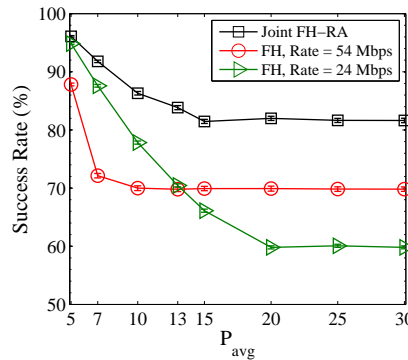
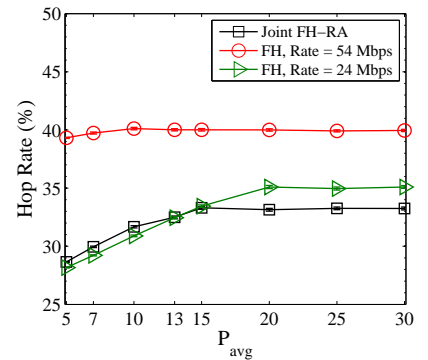
## 7 CONCLUSIONS AND FUTURE RESEARCH

In this paper, we analyzed a joint FH and RA defense scheme against a reactive-sweep Jx. We modeled the interactions between the Tx and Jx as a zero-sum Markov game, and derived the optimal equilibrium defense strategy against the worst attack strategy. Our numerical results show that the new scheme provides significant improvements in the average throughput and jamming resiliency, especially when the number of channels is small and/or the hopping cost is high. For instance, when the number of channels is between 3 and 6: (i) the throughput gain achieved by the new scheme ranges from  $\sim 15x$  to  $\sim 1.37x$ , and (ii) the improvement in the success rate varies from  $\sim 1.7x$  to  $\sim 1.13x$ .

As a future research, we aim to study the case when the Jx can also listen to the Tx's messages. In this case, the Jx can adopt the following strategy. It senses the channel for a short period at the beginning of each time slot and emits power only if a Tx activity is detected; otherwise, it does not emit any power. Because the Jx conserves power by not attacking the Tx in each time slot, it can use all of its power when it detects the Tx in a slot and cause maximum damage (i.e., the Jx emits either no power or emits maximum power depending on whether it detects Tx activity in a slot).

We also plan to analyze the joint FH and RA game when carrier aggregation is enabled (i.e., the Tx can communicate over multiple channels in each time slot). In this case, in addition to optimally deciding to hop or stay and which transmission rate to use, the Tx



Fig. 16: Average throughput vs.  $P_{avg}$ .Fig. 17: Success rate vs.  $P_{avg}$ .Fig. 18: Hop rate vs.  $P_{avg}$ .

needs to optimally select the number of channels to be aggregated in each slot. Aggregating more channels leads to higher rates if transmission is successful, but also increases the probability of being hit by the sweep  $J_x$ .

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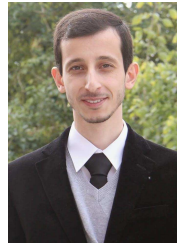
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