Energy-Aware Cooperative Wireless Networks With Multiple Cognitive Users

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Abstract—In this paper, we study and analyze cooperative 1 cognitive radio networks with arbitrary number of secondary 2 users (SUs). Each SU is considered a prospective relay for the 3 primary user (PU) besides having its own data transmission 4 demand. We consider a multi-packet transmission framework 5 that allows multiple SUs to transmit simultaneously because of 6 dirty-paper coding. We propose power allocation and scheduling 7 policies that optimize the throughput for both PU and SU with 8 minimum energy expenditure. The performance of the system 9 is evaluated in terms of throughput and delay under different 10 opportunistic relay selection policies. Toward this objective, 11 we present a mathematical framework for deriving stability con-12 ditions for all queues in the system. Consequently, the throughput 13 of both primary and secondary links is quantified. Furthermore, 14 a moment generating function approach is employed to derive a 15 closed-form expression for the average delay encountered by the 16 PU packets. Results reveal that we achieve better performance in 17 terms of throughput and delay at lower energy cost as compared 18 with equal power allocation schemes proposed earlier in the 19 literature. Extensive simulations are conducted to validate our 20 21 theoretical findings.

Index Terms—Cognitive relaying, opportunistic communica tion, throughput, delay, relay selection.

I. INTRODUCTION

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OGNITIVE radio networks have emerged as an efficient solution to the problem of spectrum scarcity and its under-utilization. In a cognitive radio network, the secondary

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users (SUs) exploit primary users' (PUs) period of inactivity 28 to enhance their performance provided that PUs' performance 29 remains unaffected. Depending on the mode of interaction 30 of the primary and the secondary users, the cognitive radio 31 networks are classified as interweave, underlay and overlay 32 networks. In the last decade or so, the industry and academia 33 has shown overwhelming interest in the application of cogni-34 tive radios in different networking solutions. Reference [2] 35 provides a comprehensive overview of the cognitive radio 36 fundamentals and research activities. 37

On the other hand, cooperative diversity has been widely 38 investigated in pursuit of combating multipath fading [3], [4]. 39 Incorporating cooperation into cognitive radio networks results 40 in substantial performance gains in terms of throughput and 41 delay for both primary and secondary nodes [5]. The SUs 42 help the PUs to transmit their data, and create opportu-43 nities for their own data transmission at the same time. 44 The cooperation between the PUs and the SUs vary from 45 just sharing information about queue states, channel state 46 information (CSI), and primary packet transmission activity 47 to the use of SUs as cognitive relays. Typically, relaying is 48 carried out over orthogonal channels due to the half-duplex 49 communication constraint at the relays [3]. However, some of 50 the recent solutions overcome this limitation by accommodat-51 ing simultaneous transmissions in a single slot [6]–[8]. This 52 is achieved through space-time coding [6] or dirty-paper 53 coding (DPC) [7], [8]. Conventionally, zero forcing and more 54 recently prior zero forcing [9] has been employed to mitigate 55 the SU signal interference with the PU signals. On the other 56 side, for cooperative cognitive radio networks with multiple 57 SUs with their own data transmission demands, employing 58 DPC allows one SU to transmit new data while the other SU 59 helps the PU by relaying its data. Thus, the spectral efficiency 60 of the system is enhanced. 61

In literature, there is a rich volume of recent work focusing 62 on cooperation in cognitive relay networks. The benefits 63 of cooperative relaying has been discussed and analyzed 64 in [10]–[12]. In [10], authors derive the maximum sustained 65 throughput of a single SU to maintain a fixed throughput 66 for PU with and without relaying. They used a dominant 67 system approach to guarantee the queue stability of both SU 68 and PU while overcoming the queues interaction. A cognitive 69 system comprising a single PU and multiple SUs along with 70 multiple relays is considered in [12], where a proportion of 71 the secondary relays help the PU in communication while a 72 relay selection is performed from the remaining relays to give 73 simultaneous access to the SU. The authors show that there 74

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exists an optimal number of cooperating relays with the PU 75 that achieve optimal outage performance. In [13], the authors 76 also discuss a cognitive relay selection problem using optimal 77 stopping theory. Reference [14] addresses a cognitive radio 78 cooperation model where the SU can transmit its data along 79 with primary transmission, but cooperates by deferring its 80 transmission when the PU is transmitting. The authors in [15] 81 address a cooperative cognitive relay network where both 82 primary and secondary nodes use cognitive relays for data 83 transmission. The relays help the PUs empty their queues 84 fast and thereby, the throughput for the SUs increases as a 85 result. SU throughput stability regions for cooperative cogni-86 tive networks have been derived for cooperative cognitive radio 87 networks in different settings in [9] and [15]. Reference [17] 88 investigates the energy efficiency in cognitive radio networks 89 via developing low-complexity algorithms for solving a joint 90 optimization problem of the spectrum sensing duration and the 91 transmit power of the cognitive users. 92

Krikidis et al. address different protocols for a cognitive 93 cooperative network and the stable throughput for both pri-94 mary and the secondary networks is derived. In this paper, 95 we adopt the model presented in [7] and employ DPC. 96 We consider a cognitive network with arbitrary number of 97 SUs co-existing with a PU and sharing one common relay 98 queue. We propose power allocation and scheduling poli-99 cies that enhance the throughput of both primary and sec-100 ondary links using the least possible energy expenditure. 101 The summary of the main contributions of this work is as 102 follows. 103

- We propose an energy-efficient adaptive power (AP) allocation scheme for the SUs that enhances the throughput of both primary and secondary links. Energy-efficient transmission is achieved via exploiting instantaneous CSI to adapt the transmission powers at all SUs.
- We introduce two SU scheduling policies, which prioritize primary or secondary throughput enhancement according to the network requirements. We analyze the performance of both policies in conjunction with equal and adaptive power allocation schemes.

We develop a generic mathematical framework to derive 114 closed-form expressions for both PU and SU throughput, 115 and PU average delay. The mathematical analysis is 116 performed for an arbitrary number of SUs coexisting with 117 a PU. A detailed analysis is performed for each combi-118 nation of power allocation and SU scheduling policies. 119 We validate our theoretical findings via simulations. 120 Results reveal that AP-based schemes yield superior 121 performance compared to EP allocation proposed in [7], 122 with significantly less energy cost. 123

The rest of this paper is organized as follows. Section II 124 presents the information-theoretic background and preliminar-125 ies needed in the sequel. Section III introduces the system 126 model and the proposed cooperation strategy. The opportunis-127 tic relay selection and power allocation strategies are presented 128 in Section IV along with their mathematical analysis in 129 Section V. Numerical results are then presented in Section VI. 130 Finally, concluding remarks are drawn in Section VII. 131

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Fig. 1. Cognitive radio network model under consideration. The (logical) CSB is shown to coordinate the activities of the common relay queue.

II. BACKGROUND AND PRELIMINARIES

A. Dirty-Paper Coding

DPC was first introduced in [18] and we briefly state its 134 implication. Consider a channel with output $\mathbf{y} = \mathbf{x} + \mathbf{q} + \mathbf{z}$, 135 where \mathbf{x} , \mathbf{q} and \mathbf{z} denote the input, interference, and noise, 136 respectively. The input $\mathbf{x} \in \mathbb{C}^m$ satisfies the power con-137 straint $(1/m) \sum_{i=1}^{m} |x_i|^2 \le P_0$. We assume that **q** and **z** are 138 zero-mean Gaussian vectors with covariance matrices QI_m 139 and $N_0 \mathbf{I}_m$, respectively, where \mathbf{I}_m denotes the $m \times m$ identity 140 matrix. If the interference \mathbf{q} is unknown to both transmitter and 141 receiver, the channel capacity is given by $\log(1+P_0/(Q+N_0))$ 142 (bits/channel use). However, if \mathbf{q} is known to the transmitter 143 but not the receiver, the channel capacity is shown to be 144 the same as that of a standard "interference free" Gaussian 145 channel with signal-to-noise ratio P_0/N_0 using DPC. In other 146 words, if the interference is known a priori at the transmitter, 147 DPC renders the link between the transmitter and its intended 148 receiver interference-free. 149

B. Channel Outage

We present the basic definition of an outage event and 151 the corresponding outage probability calculation. Consider a 152 channel with output $\mathbf{y} = \sqrt{\mathbf{h}}\mathbf{x} + \mathbf{z}$, where $\sqrt{\mathbf{h}}$ and \mathbf{x} denote 153 the fading coefficient and the input, respectively. Moreover, 154 the noise z is modelled as zero-mean circularly symmetric 155 complex Gaussian random variable with variance N_0 . For a 156 target transmission rate R_0 , an outage occurs if the mutual 157 information between the input and output is not sufficient to 158 support that rate. The probability of such event, for a channel 159 with average power constraint P_0 , is 160

$$\mathbb{P}\left[\mathbf{h} < \frac{2^{R_0} - 1}{P_0/N_0}\right].$$
 (1) 16

III. System Model

We consider the cognitive radio system shown in Fig. 1. 163 The system comprises a PU p that transmits its packets to a 164 primary destination D_p . A cognitive network consisting of an 165 arbitrary number of SUs coexists with the primary network. 166 The number of SUs is denoted by N and we refer to the set of 167 SUs by $\mathbb{S} = \{s_i\}_{i=1}^N$. Each SU has its own data that requires 168 to be delivered to a common secondary destination D_s . All 169 nodes are equipped with infinite capacity buffers. Time is 170

slotted, and the transmission of a packet takes exactly one time 171 slot. The duration of a time slot is normalized to unity and 172 hence, the terms power and energy are used interchangeably 173 in the sequel. We take into account the bursty nature of the 174 source through modelling the arrivals at the PU as a Bernoulli 175 process with rate λ_p (packets/slot). In other words, at any 176 given time slot, a packet arrives at the PU with probability 177 $\lambda_p < 1$. The arrival process at the PU is independent and 178 identically distributed (i.i.d.) across time slots. On the other 179 hand, the SUs are assumed backlogged, i.e., SUs always 180 have packets awaiting transmission. We assume that the SUs 181 perfectly sense the PU's activity, i.e., there is no chance of 182 collision between the PU and any of the secondary users. 183 A node that successfully receives a packet broadcasts an 184 acknowledgment (ACK) declaring the successful reception 185 of that packet. ACKs sent by the destinations are assumed 186 instantaneous and heard by all nodes error-free. 187

The channel between every transmitter-receiver pair exhibits 188 frequency-flat Rayleigh block fading, i.e., the channel coeffi-189 cient remains constant for one time slot and changes indepen-190 dently from one slot to another. The scalars $\mathbf{h}_{r_i}[n]$ and $\mathbf{h}_{s_i}[n]$ 191 denote the absolute squared fading coefficient of the channels 192 that connect the *i*th SU to D_p and D_s , respectively, at the *n*th 193 time slot. Similarly, the absolute squared fading coefficient of 194 the channels that connect the PU to D_p and s_i , at the *n*th time 195 slot, are denoted by $\mathbf{h}_p[n]$ and $\mathbf{h}_{ps_i}[n]$, respectively. According 196 to the Rayleigh fading assumption, $\mathbf{h}_{r_i}[n]$, $\mathbf{h}_{s_i}[n]$, and $\mathbf{h}_{ps_i}[n]$ 197 are exponential random variables with means σ^2 , for all 198 $i = 1, \ldots, N$. We denote an exponential random variable 199 with mean σ^2 by $\exp(\sigma^2)$. Then, we have $\mathbf{h}_p[n] \sim \exp(\sigma_p^2)$. 200 All links are considered statistically equivalent except for the link $p \rightarrow D_p$. We assume that $\sigma_p^2 < \sigma^2$ to demonstrate the 201 202 benefits of cooperation [19]. For the ease of exposition, we set 203 $\sigma^2 = 1$ throughout the paper. All communications are subject 204 to additive white Gaussian noise of variance N_0 . 205

Next, we present the queuing model of the system followed 206 by the description of the employed cooperation strategy. 207

A. Queuing Model 208

The queues involved in the system analysis, shown in Fig. 1, 209 are described as follows: 210

- Q_p : a queue that stores the packets of the PU correspond-211 ing to the external Bernoulli arrival process with rate λ_p . 212
- Q_{s_i} : a queue that stores the packets at the *i*th SU, where 213 $i \in \{1,\ldots,N\}.$ 214

• Q_r : a queue that stores PU packets to be relayed to D_p . 215 Having independent relay queues for all SUs makes exact 216 performance analysis intractable with the increasing number 217 of users. To address this complexity, Krikidis et al. introduced 218 the idea of a common 'fictitious' relay queue Q_r in [7], which 219 is maintained by a so-called cluster supervision block (CSB) 220 that controls and synchronizes all the activities of the cognitive 221 cluster. Along the lines of [7], we assume the existence 222 of a common relay such that SUs can perfectly exchange 223 information with the CSB with a negligible overhead. The 224 channels $\mathbb{S} \to D_p, D_s$ are assumed known instantaneously at 225 the CSB [7], [20]. 226

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The instantaneous evolution of queue lengths is captured as 227

$$\mathbf{Q}_{i}[n+1] = (\mathbf{Q}_{i}[n] - \mathbf{L}_{i}[n])^{+} + \mathbf{A}_{i}[n], \quad i \in \{p, r\} \cup \mathbb{S} \quad (2) \quad {}_{228}$$

where $(x)^+ = \max(x, 0)$ and $\mathbf{Q}_i[n]$ denotes the number of 229 packets in the *i*th queue at the beginning of the *n*th time slot. 230 The binary random variables taking values either 0 or 1, $L_i[n]$ 231 and $A_i[n]$, denote the departures and arrivals corresponding to 232 the *i*th queue in the *n*th time slot, respectively. 233

B. Cooperation Strategy

The employed cooperative scheme is described as follows.

- 1) The PU transmits a packet whenever Q_p is non-empty. 236
- 2) If the packet is successfully decoded by D_p , it broadcasts an ACK and the packet is dropped from Q_p .
- 3) If the packet is not successfully received by D_p yet successfully decoded by at least one SU, an ACK is broadcasted and the packet is buffered in Q_r and dropped from Q_p .
- 4) If D_p and S fail to decode the packet, it is kept at Q_p 243 for retransmission in the next time slot.
- 5) When the PU is sensed idle, if Q_r is non-empty, two out 245 of all SUs transmit simultaneously. One SU is selected 246 to relay a packet from Q_r to D_p and is denoted by r^* . 247 Another SU is selected to transmit a packet of its own 248 to D_s and is denoted by s^* . Otherwise, if Q_r is empty, 249 one SU is selected to transmit a packet to D_s .¹ The SUs' 250 selection policies are explained in Section IV-B.
- If the packets transmitted by the SUs are successfully 6) 252 received by their respective destinations, ACKs are 253 broadcasted and these packets exit the system. Other-254 wise, the packet that experiences unsuccessful transmis-255 sion is kept at its queue for later retransmission. 256

IV. POWER ALLOCATION AND NODE SELECTION

In this section, we introduce the adaptive power allocation 258 and opportunistic relay selection strategies for an arbitrary 259 number of SUs, $N \ge 2$. We propose a power allocation policy 260 that minimizes energy consumption at each SU as compared 261 to a fixed power allocation policy in [7]. In the sequel, node 262 selection policy refers to the choice of the SU that relays a pri-263 mary packet from Q_r to D_p , and the SU that transmits a packet 264 from its own queue to D_s , i.e., the selection of r^* and s^* . 265 The availability of CSI for all the channels (and thereby 266 incurred interference) at the CSB is exploited to perform power 267 allocation and node selection online, i.e., every time slot. 268

A. Power Allocation

Whenever Q_p is non-empty, the PU transmits a packet 270 with average power P_0 . However, when the PU is idle and 271 Q_r is non-empty, two SUs out of N transmit simultane-272 ously by employing DPC [18]. One SU relays a primary 273 packet to D_p while the other transmits a secondary packet 274 to D_s . Since all SUs can perfectly exchange information with 275

¹Note that two SUs can be selected for transmission if Q_r is empty. However, this requires multi-packet reception capability at the secondary destination which is out of the scope of this paper.

the CSB, Q_r is accessible by both SUs selected for transmis-276 sion. Therefore, the transmission of r^* is considered a priori 277 known interference at s^* . Accordingly, s^* adapts its signal 278 to see an interference-free link to D_s using the result stated 279 in Section II-A. On the other hand, s^* transmits a packet 280 from its own queue which is not accessible by r^* . Thus, 281 the transmission of s^* causes an interference on the relay 282 link, i.e., $r^* \rightarrow D_p$. The achievable rate region on this 283 Z-interference channel at the *n*th time slot is given by 284

$$\mathbf{R}_{s^*}[n] = \log \left[1 + \frac{P_{s^*}[n]\mathbf{h}_{s^*}[n]}{N_0} \right]$$
(3)

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$$\mathbf{R}_{r^*}[n] = \log\left[1 + \frac{P_{r^*}[n]\mathbf{h}_{r^*}[n]}{N_0 + P_{s^*}[n]\mathbf{h}_{\mathrm{I}}[n]}\right]$$
(4)

where $P_{s*}[n]$ and $P_{r*}[n]$ denote the instantaneous trans-287 mit powers of s^* and r^* , respectively. The instantaneous 288 absolute squared fading coefficients of the secondary, relay, 289 and interference links are denoted by $\mathbf{h}_{s*}[n]$, $\mathbf{h}_{r*}[n]$, and $\mathbf{h}_{I}[n]$, 290 respectively. We denote the links $s^* \rightarrow D_s$, $r^* \rightarrow D_p$, 291 and $s^* \to D_p$ by the secondary, the relay, and the interference 292 link, respectively. Hereafter, we omit the temporal index 293 *n* for simplicity. Nevertheless, it is implicitly understood that 294 power allocation and node selection are done on a slot-by-295 slot basis. In this work, we focus on developing an adaptive 296 power allocation scheme for the transmitting SUs that use 297 a fixed transmission rate R_0 . Specifically, our multi-criterion 298 objective is to enhance primary and secondary throughput 299 while minimizing the energy consumption at each SU. The 300 rates given by (3) and (4) stimulate thinking about how power 301 is allocated to both transmitting SUs. 302

Next, we investigate two different power allocation policies 303 for the SUs, namely, equal power (EP) allocation and adaptive 304 power (AP) allocation. It is worth noting that power allocation 305 and node selection are performed for the SUs since we have 306 no control on the PU. Thus, in the following lines, we focus 307 on the slots in which the PU is idle. 308

1) Equal Power Allocation: This policy assigns equal trans-309 mission powers to the SUs as proposed in [7] and serves as 310 a baseline scheme in this work. Whenever an SU transmits, 311 it uses an average power P_{max} . Specifically, if an SU is 312 transmitting alone, e.g., Q_r is empty, it uses a power P_{max} . 313 If two SUs transmit simultaneously, e.g., Q_r is non-empty, 314 $P_{s^*} = P_{r^*} = P_{\max}.$ 315

2) Adaptive Power Allocation: Unlike EP allocation, 316 we exploit the CSI available at the CSB to propose an AP 317 allocation scheme that minimizes the average power consump-318 tion at each SU. We use (3) and (4) along with (1) to derive 319 conditions on P_{s^*} and P_{r^*} for successful transmission at a 320 target transmission rate R_0 . These conditions are 321

$$P_{s^*} \ge \frac{(2^{R_0} - 1)N_0}{\mathbf{h}_{s^*}} \tag{5}$$

(6)

$$P_{r^*} \geq \frac{(2^{R_0} - 1)[N_0 + P_{s^*} \mathbf{h}_I]}{\mathbf{h}_{r^*}}.$$

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A transmitter that violates the condition on its transmis-324 sion power experiences a sure outage event. Furthermore, 325 we impose a maximum power constraint at each SU, where 326 $P_{s^*}, P_{r^*} \leq P_{\max}$. It is worth noting that P_{s^*} is computed first 327

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according to (5) followed by the computation of P_{r^*} according 328 to (6). In a given slot, if P_{max} is less than the power required to 329 guarantee a successful transmission for a given SU, i.e., P_{max} 330 is less than the right hand sides of either (5) or (6), the CSB 331 sets the power of that SU to zero to avoid a guaranteed outage 332 event. Clearly, this results in increasing the throughput of the 333 PU due to reduction in the amount of interference caused 334 by the transmission of s^* on the relay link in the time slots 335 where s* refrains from transmitting. Moreover, compared to 336 EP allocation, energy wasted in slots where a sure outage event 337 occurs is now saved. 338

B. Node Selection Policies

We consider a system that assigns full priority to the 340 PU to transmit whenever it has packets. Therefore, the SUs 341 continuously monitor the PU's activity seeking an idle time 342 slot. When the PU is sensed idle, the SUs are allowed 343 to transmit their own and/or a packet from the common 344 queue Q_r . Note that it is possible to transmit only one packet 345 by the SUs in the following scenarios: 346

- 1) If Q_r is empty, i.e., no primary packet to be relayed. 347 Then, we select the SU with the best channel to D_s .
- 2) Q_r is non-empty, but r^* or s^* is set silent by the 349 CSB to avoid a guaranteed outage event on the $r^* \rightarrow$ 350 D_p or $s^* \to D_s$ link. Note that CSI for transmission is 351 assumed to be known at CSB and outage event (due to 352 power limitation) can be predicted before transmission 353 as discussed in Section IV-A.2. In this case, we choose 354 the transmitting SU as the one with the best instanta-355 neous link to the intended destination. For example, if r^* 356 is silent and s^* is transmitting alone, the SU with the 357 best link between $\mathbb{S} \to D_s$ transmits. 358

The case for the simultaneous transmission of two SUs is 359 the main topic for investigation in this paper. If the two 360 transmissions occur simultaneously, the transmitting SUs are 361 selected according to one of the following policies. 362

1) Best Secondary Link (BSL): In this policy, the utility 363 function to be maximized is the SU throughput. Therefore, 364 we choose the SU that transmits a packet of its own as the 365 one with the best instantaneous link to D_s , i.e., 366

$$\mathbf{h}_{S^*} = \max_{i \in \{1, \dots, N\}} \mathbf{h}_{s_i}.$$
 (7) 36

Among the remaining (N-1) SUs, the one with the best 368 instantaneous link to D_p is chosen to be r^* . 369

2) Best Primary Link (BPL): In this policy, unlike BSL, 370 the utility function to be maximized is PU throughput. Thus, 371 we choose the SU that relays a primary packet from Q_r as 372 the one with the best instantaneous link to D_p , i.e., 373

$$\mathbf{h}_{r^*} = \max_{i \in \{1, \dots, N\}} \mathbf{h}_{r_i}.$$
 (8) 374

Among the remaining (N-1) SUs, the one with the best 375 instantaneous link to D_s is chosen to be s^* . 376

It is worth noting that all links $\mathbb{S} \rightarrow D_p, D_s$ are sta-377 tistically independent. Thus, at any given time slot, if a 378 certain SU has the best instantaneous channel to a cer-379 tain destination, e.g., D_p , we can not infer any infor-380 mation about its link quality to the other destination, 381 e.g., D_s . Hence, $\forall i \in \{1, ..., N\}$, s_i can have the best link to D_p/D_s irrespective of the quality of its link to the other destination.

So far, we have introduced two policies for each of the power allocation and SU scheduling policies. Thus, we have four different cases arising from the possible combinations of these policies. Next, we proceed with the performance analysis of the system for each case.

V. THROUGHPUT AND DELAY ANALYSIS

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In this section, we conduct a detailed analysis for the system 391 performance in terms of throughput and delay. Towards this 392 objective, we derive the stability conditions on the queues with 393 stochastic packet arrivals, namely, Q_p and Q_r . The stability 394 of a queue is loosely defined as having a bounded queue size, 395 i.e., the number of packets in the queue does not grow to infin-396 ity [19]. Furthermore, we analyze the average queuing delay 397 of the primary packets. We obtain a closed-form expression 398 for this delay through deriving the moment generating func-399 tion (MGF) of the joint lengths of Q_p and Q_r . It is worth not-400 ing that the SUs' queues are assumed backlogged and hence, 401 no queueing delay analysis is performed for the secondary 402 packets. In the following lines, we provide a general result for 403 the throughput of the primary and secondary links as well as 404 the delay of primary packets. Then, we proceed to highlight 405 the role of the proposed power allocation and node selection 406 policies. We first introduce some notation. The probabilities of 407 successful transmissions on the relay and secondary links are 408 denoted by f_{r^*} and f_{s^*} , respectively. A transmission on the 409 link $p \rightarrow D_p$ is successful with probability f_p . In addition, 410 the probability that at least one SU successfully decodes a 411 transmitted primary packet is denoted by f_{ps} . 412

Theorem 1: The maximum achievable PU throughput for
the system shown in Fig. 1, under any combination of power
allocation and node selection policies, is given by

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$$\lambda_p < \frac{f_{r^*}[f_p + (1 - f_p)f_{ps}]}{f_{r^*} + (1 - f_p)f_{ps}}$$
(9)

417 while the throughput of the SU $s_i \in S$ is given by

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$$\mu_{s_i} = \frac{1}{N} \left[1 - \frac{\lambda_p}{f_p + (1 - f_p) f_{ps}} \right] f_{s^*}.$$
 (10)

⁴¹⁹ *Proof:* We use Loynes' theorem [21] to establish the ⁴²⁰ stability conditions for Q_p and Q_r . The theorem states that ⁴²¹ if the arrival and service processes of a queue are stationary, ⁴²² then the queue is stable if and only if the arrival rate is strictly ⁴²³ less than the service rate. Therefore, for Q_p to be stable, ⁴²⁴ the following condition must be satisfied

$$\lambda_p < \mu_p \tag{11}$$

where μ_p denotes the service rate of Q_p . A packet departs Q_p if it is successfully decoded by at least one node in $\mathbb{S} \cup \{D_p\}$. Thus, μ_p is given by

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$$\mu_p = f_p + (1 - f_p) f_{ps}.$$
 (12)

430 Similarly, Q_r is stable if

$$\frac{\lambda_p}{\mu_p}(1-f_p)f_{ps} < \left[1-\frac{\lambda_p}{\mu_p}\right]f_{r^*}.$$
(13)

A PU's packet arrives at Q_r if Q_p is non-empty and an 432 outage occurs on the direct link $p \rightarrow D_p$ yet no outage 433 occurs at least on one link between $p \rightarrow S$. From Little's 434 theorem [22], we know that probability of Q_p being non-435 empty equals λ_p/μ_p . This explains the rate of packet arrivals 436 at Q_r shown on the left hand side (LHS) of (13). The right 437 hand side (RHS) represents the service rate of Q_r . A packet 438 departs Q_r if Q_p is empty and there is no outage on the 439 link $r^* \to D_p$. Rearranging the terms of (13), we obtain the 440 maximum achievable PU throughput as given by (9) provided 441 that μ_p is given by (12). It is worth noting that (9) provides 442 a tighter bound on λ_p than (11) due to the multiplication of 443 μ_p in (9) by a term less than one. 444

On the other hand, we compute the throughput of SUs 445 by calculating the service rate of their queues since they 446 are assumed backlogged. Due to the symmetric configura-447 tion considered, i.e., statistically equivalent links $\mathbb{S} \to D_s$, 448 the throughput of all SUs is the same. For $s_i \in S$, a packet 449 departs Q_{s_i} if Q_p is empty, s_i is selected to transmit a packet 450 of its own and no outage occurs on the link $s_i \rightarrow D_s$. 451 Due to symmetry, at any time slot, all SUs have equal 452 probabilities to be selected to transmit a packet from their own 453 queues, i.e., $\mathbb{P}[s^* = s_i] = 1/N \ \forall i \in \{1, \dots, N\}$. Therefore, 454 the SUs' throughput is given by (10) provided that μ_p is 455 given by (12). \Box 456

Next, we develop a mathematical framework to analyze the average queuing delay for the PU's packets.

Theorem 2: The average queuing delay encountered by the459PU packets in the system shown in Fig. 1, under any combi-
nation of power allocation and node selection policies, is460

$$\tau = \frac{N_p + N_r}{\lambda_p} \tag{14}$$

where N_p and N_r , the average lengths of Q_p and Q_r , 463 respectively, are given by 464

$$N_p = \frac{-\lambda_p^2 + \lambda_p}{\mu_p - \lambda_p} \tag{15}$$

$$N_r = \frac{r\lambda_p^2 + s\lambda_p}{\delta\lambda_p^2 + \zeta\lambda_p + \eta} \tag{16}$$

and

$$r = f_{ps}(1 - f_p) \left[\frac{f_{r^*} - f_p}{\mu_p} - f_{r^*} - f_{ps}(1 - f_p) \right]$$
(17) 466

$$\delta = f_{ps}(1 - f_p)\mu_p$$
(18) 469
$$\delta = f_{rs} + f_{rs}(1 - f_s)$$
(19) 470

$$\zeta = \mu_p \left[-2f_{r^*} - f_{ps}(1 - f_p) \right]$$
(19) 470
$$\zeta = \mu_p \left[-2f_{r^*} - f_{ps}(1 - f_p) \right]$$
(20) 471

$$\eta = \mu_p^2 f_{r^*} \tag{21} \quad {}_{472}$$

while μ_p is given by (12).

Proof: If a primary packet is directly delivered to D_p , 474 it experiences the queuing delay at Q_p only. This happens 475 with a probability $1 - \epsilon = f_p / \mu_p$. However, if the packet is 476 forwarded to D_p through the relay link, it experiences the total 477 queuing delay at both Q_p and Q_r . Thus, the average delay is 478

$$\tau = (1 - \epsilon)\tau_p + \epsilon(\tau_p + \tau_r) = \tau_p + \epsilon\tau_r \tag{22}$$

457

458

where τ_p and τ_r denote the average delays at Q_p and Q_r , 480 respectively. The arrival rates at Q_p and Q_r are given by λ_p 481 and $\epsilon \lambda_p$, respectively. Thus, applying Little's law [22] renders 482

$$\tau_p = N_p / \lambda_p, \quad \tau_r = N_r / \epsilon \lambda_p. \tag{23}$$

Substituting (23) in (22) renders τ exactly matching (14). 484

Proceeding with computing N_p , we make use of the fact 485 that Q_p is a discrete-time M/M/1 queue with arrival rate λ_p 486 and service rate μ_p . Thus, N_p is directly given by (15) through 487 applying the Pollaczek-Khinchine formula [23]. However, the 488 dependence of the arrival and service processes of Q_r on 489 the state of Q_p necessitates using a MGF approach [24] to 490 calculate N_r . The MGF of the joint lengths of Q_p and Q_r is 491 defined as 492

$$G(x, y) = \lim_{n \to \infty} \mathbb{E} \left[x^{\mathbf{Q}_p[n]} y^{\mathbf{Q}_r[n]} \right]$$
(24)

where \mathbb{E} denotes the statistical expectation operator. Following 494 the framework in [4] and [24], we get 495

496
$$G(x, y) = (\lambda_p x + 1 - \lambda_p) \frac{B(x, y)G(0, 0) + C(x, y)G(0, y)}{yD(x, y)}$$
497 (25)

where 498

$$B(x, y) = x(y-1)f_{r^{*}} C(x, y) = xf_{r^{*}} - yf_{p} - y^{2}f_{ps}(1-f_{p}) + xy(\mu_{p} - f_{r^{*}}) D(x, y) = x - (\lambda_{p}x + 1 - \lambda_{p})[f_{p} + yf_{ps}(1-f_{p}) + x(1-\mu_{p})].$$

$$(26)$$

First, we compute the derivative of (25) with respect to y and 503 then, take the limit of the result when x and y tend to 1. This 504 verifies that N_r is given by (16). 505

Theorems 1 and 2 provide closed-form expressions for the 506 network performance metrics, throughput and delay. These 507 expressions are mainly functions of the outage probabilities 508 on various links in the network, namely, f_p , f_{ps} , f_{r*} , and f_{s*} . 509 In the following lines, we quantify these outage probabilities 510 for the different combinations of power allocation and node 511 selection policies. It is worth noting that f_p and f_{ps} are 512 related to the PU side. Therefore, they remain the same for all 513 combinations of power allocation and node selection policies 514 which are performed at the SUs side. Using (1), we have 515

516
$$f_p = \mathbb{P}\left[\mathbf{h}_p > \frac{2^{R_0} - 1}{P_0/N_0}\right] = e^{-\alpha/\sigma_p^2}$$
(27)

where $\alpha = \frac{2^{R_0}-1}{P_0/N_0}$. This follows from the Rayleigh fading assumption that renders $\mathbf{h}_p \sim \exp(\sigma_p^2)$. Similarly, 517 518

519
$$f_{ps} = \mathbb{P}\left[\max_{i \in \{1, \dots, N\}} \mathbf{h}_{ps_i} > \alpha\right] = 1 - (1 - e^{-\alpha})^N. \quad (28)$$

On the other hand, we shift our attention to the SU side to 520 calculate f_{r^*} and f_{s^*} . We analyze the four cases arising from 521 the proposed power allocation and relay selection policies in 522 the following order: (i) EP-BSL, (ii) EP-BPL, (iii) AP-BSL, 523 and (iv) AP-BPL. Towards this objective, we first note that 524 for each SU, its link qualities to D_p and D_s are statistically 525 independent. Furthermore, these links are independent of the 526

other (N-1) users' links. Thus, we are dealing with 2N 527 i.i.d. random variables, \mathbf{h}_{r_i} and \mathbf{h}_{s_i} , $\forall i \in \{1, \dots, N\}$. Each 528 of these variables is exponentially distributed with mean 1 as 529 a direct consequence of the Rayleigh fading model consid-530 ered. We begin with an analysis of the distributions of the 531 random variables involved in the derivations of f_{r^*} and f_{s^*} , 532 specifically, \mathbf{h}_{r^*} , \mathbf{h}_{I} , and \mathbf{h}_{s^*} . Finding these distributions is 533 fundamental to the mathematical derivations presented next. 534 Obviously, the distributions is dependent on the node selection 535 policy employed and hence, we present a separate analysis for 536 BSL and BPL in Appendices A and B, respectively. 537

For the ease of exposition, we define $a = \frac{2^{R_0} - 1}{P_{\text{max}}/N_0}$, $b = (2^{R_0} - 1)^{-1}$, and $\beta = 1 - e^{-a}$. The exponential integral 538 539 function, $E_1[.]$, is defined as $E_1[x] = \int_x^\infty (e^{-t}/t) dt$. 540 541

Lemma 1: For EP-BSL, f_{r^*} and f_{s^*} are given by

$$f_{r^*} = 1 - \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k \frac{e^{-ka}}{(1+k/b)}$$
(29) 542
$$f_{r^*} = 1 - \frac{R^N}{k}$$
(30) 542

$$f_{s^*} = 1 - \beta^{r_*}.$$
 (30) 543

Proof: See Appendix C. 544

Lemma 2: For EP-BPL, f_{r^*} is given by

$$f_{r^*} = \frac{N}{N-1} \sum_{k=1}^{N-1} \binom{N-1}{k-1} [I_1 - I_2].$$
(31) 546

where

$$I_{1} = \sum_{m=0}^{k-1} {\binom{k-1}{m}} \frac{(-1)^{m}}{(N-k+m+1)}$$
(32) 54

$$I_{2} = \sum_{m=0}^{k-1} \sum_{\ell=0}^{N} {\binom{k-1}{m} \binom{N}{\ell} \frac{(-1)^{m+\ell} e^{-a\ell}}{(N-k+m+\ell/b+1)}}$$
(33) 545

On the other hand, f_{s^*} is given by

$$f_{s^*} = \gamma \left(1 - \beta^{N-1} \right) + (1 - \gamma) \left(1 - \beta^N \right)$$
 (34) 551

where

$$\gamma = \frac{\lambda_p (1 - f_p) f_{ps}}{(\mu_p - \lambda_p) f_{r^*}}.$$
(35) 55

Proof: See Appendix D. Lemma 3: For AP-BSL, f_{r^*} is given by

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$$f_{r^*} = \beta^N (1 - \beta^N) + N \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k$$
556

$$\times e^{-a(k+1)} [I_3 - I_4]$$
 (36) 557

where

$$I_{3} = \frac{N-1}{k+1} \sum_{\ell=0}^{N-2} {\binom{N-2}{\ell} \frac{(-1)^{\ell}}{(\ell+1)}} e^{-a(\ell+1)}$$
(37) 55

$$I_{4} = \frac{a}{b}e^{ab}(N-1)\sum_{\ell=0}^{N-2} {\binom{N-2}{\ell}}(-1)^{\ell}e^{\frac{a(1+b+\ell)(k+1-b)}{b}}$$

$$[a(1+b+\ell)(k+1)]$$
560

$$\times E_1 \left\lfloor \frac{a(1+b+\ell)(k+1)}{b} \right\rfloor. \tag{38}$$

On the other hand, f_{s^*} is given by (30). 562 Proof: See Appendix E. 563



Fig. 2. The probability of transmission success on the relay link versus P_{max}/N_0 for AP-based schemes. (a) AP-BSL. (b) AP-BPL.

(39)

(43)

Lemma 4: For AP-BPL, f_{r^*} is given by 564

$$f_{r^*} = \sum_{k=1}^{N-1} \sum_{\ell=0}^{k-1} \sum_{m=0}^{N-2} \binom{N-1}{k-1} \binom{k-1}{\ell} \binom{N-2}{m} \times \frac{(-1)^{m+\ell} N^2 [I_5 - I_6]}{(N-k+\ell+1)} + \beta^{N-1} (1-\beta^N)$$

$$\times \frac{(-1)^{m+\ell}}{(N-\ell)}$$

where

568 I₅ =
$$\frac{e^{-a(m+1)}}{(m+1)} \sum_{n=0}^{N-1} {\binom{N-1}{n}} \frac{(-1)^n e^{-a(n+1)}}{(n+1)}$$
 (40)
569 I₆ = $\sum_{n=0}^{N-1} {\binom{N-1}{n}} \frac{a(-1)^n e^{-a(m+n-t+2)}}{(t-n-1)} e^{tc} E_1 [t(a+c)]$
570 (41)

570

57

576

and the terms t and c are 571

572
$$t = b(N - k + \ell + 1) + n + 1$$
(42)

$$c = a \left\lfloor \frac{m+1}{b(N-k+\ell+1)} - 1 \right\rfloor.$$

On the other hand, f_{s*} is given by (34). 574 Proof: See Appendix F. 575

VI. NUMERICAL RESULTS

In this section, we validate the closed-form expressions 577 derived in the paper via comparing theoretical and numerical 578 simulation results. We investigate the system performance in 579 terms of the primary and secondary throughput as well as the 580 average primary packets' delay. In addition, we quantify the 581 average power consumption at the SUs. Furthermore, we con-582 duct performance comparisons between the four strategies 583 resulting from the proposed power allocation and SU selection 584 policies. Accordingly, we draw insights about the benefit of 585 employing the proposed power allocation schemes. We set 586 $P_0/N_0 = 10$ dB. Results are averaged over 10^6 time slots. 587

Theorems 1 and 2 provide closed-form expressions for 588 primary and secondary throughput as well as average queueing 589 delay for primary packets. Generic expressions have been 590 provided that work for any combination of power allocation 591 and node selection policies. These expressions are functions 592 of the probabilities of successful transmissions on relay and 593

secondary links, i.e., f_{r^*} and f_{s^*} . This fact has been thoroughly 594 addressed in the appendices, where the four different power 595 allocation and node selection policies have been analyzed. 596 We start by validating our theoretical findings through sim-597 ulations. Towards this objective, the analytical expressions 598 for f_{r^*} , derived in Appendix E and F, are compared to 599 their corresponding simulation results for both AP-BSL and 600 AP-BPL in Fig. 2. We set a target rate $R_0 = 1.5$ (bits/channel 601 use) and we choose $\sigma_p^2 = 0.25$. Fig. 2(a) shows a perfect 602 match of theoretical and simulation results for AP-BSL for 603 any number of SUs, N. However, for AP-BPL, Fig. 2(b) 604 shows a slight deviation between both results. This difference 605 is attributed to the relaxation of the constraint that $\mathbf{h}_{\mathrm{I}} < \mathbf{h}_{r^*}$ 606 in the derivation presented in Appendix F, where we treat \mathbf{h}_{I} 607 and \mathbf{h}_{r^*} as independent random variables. This constraint is an 608 immediate consequence of the node selection policy presented 609 in Section IV-B.2. The relaxation has been done for the sake 610 of mathematical tractability. Nevertheless, Fig. 2(b) shows that 611 the constraint relaxation has a minor effect on the obtained 612 closed-form expression for f_{r^*} . This validates our theoretical 613 findings. Fig. 2 show that f_{r^*} consistently increases as the 614 number of SUs increases for both AP-based schemes. This 615 behavior is also true for EP-based schemes and is attributed 616 to multi-user diversity gains obtained through increasing N. 617

We investigated the effect of varying N in Fig. 2. Without 618 loss of generality, the rest of the results are presented for 619 N = 2, $R_0 = 2$ (bits/channel use), and $\sigma_p^2 = 0.25$. 620 We proceed with presenting the throughput of the PU and 621 the SUs for all combinations of power allocation and node 622 selection policies in Fig. 3. In Fig. 3(a), we plot the maximum 623 achievable PU throughput, i.e., maximum achievable λ_p given 624 by (9) in Theorem 1, versus P_{max}/N_0 . AP-BPL is shown to 625 outperform all other schemes. In particular, AP-BPL increases 626 the PU's throughput by up to 30% compared to AP-BSL 627 and EP-BPL, and more than 100% compared to EP-BSL. 628 Moreover, it is evident that AP-based schemes outperform 629 EP-based schemes [7], irrespective of the node selection policy 630 employed. In Fig. 3(b), we plot the SU throughput versus 631 λ_p at $P_{\rm max}/N_0 = 7$ dB. For the same node selection policy, 632 the throughput region of the AP-based schemes is shown to 633 strictly contain that of the EP based scheme. Furthermore, 634 at every feasible λ_p for EP-BPL, higher SU throughput 635



Fig. 3. The throughput of the PU and SUs for all combinations of power allocation and node selection policies. (a) Maximum achievable PU throughput versus P_{max}/N_0 . (b) SU throughput versus λ_p .



Fig. 4. The average queueing delay of PU's packets for different combinations of power allocation and node selection policies. (a) Average primary packets' delay versus P_{max}/N_0 . (b) Average primary packets' delay versus λ_p .

is attained by AP-BPL. Thus, power adaptation expands
 the stable throughput region. This shows the superiority of
 AP-based schemes in both PU and SU throughput over their
 EP-based counterparts.

In Fig. 4, we study the average delay encountered by the PU 640 packets. We refrain from plotting the results corresponding to 641 EP-BSL to get a clear view of the comparison. EP-BSL yields 642 much worse delay than the other three strategies. We plot the 643 average primary packet delay versus P_{max}/N_0 in Fig. 4(a) 644 at $\lambda_p = 0.1$. As the available power resources increase, 645 i.e., $P_{\rm max}/N_0$ increases, delay decreases. We attain lower 646 average delay through power adaptation. As expected, AP-BPL 647 holds its position as the best scheme with respect to PU. 648 Furthermore, we investigate the fundamental throughput-delay 649 tradeoff in Fig. 4(b). We plot the average packet delay for the 650 PU versus its throughput at $P_{\text{max}}/N_0 = 5$ dB. Intuitively, when 651 a node needs to maintain a higher throughput, it loses in terms 652 of the average delay encountered by its packets. Given that the 653 system is stable, the node's throughput equals its packet arrival 654 rate. Thus, increased throughput means injecting more packets 655 into the system resulting in a higher delay. Furthermore, 656 Fig. 4(b) shows that strictly lower average PU delay is attained 657 via AP-based schemes compared to EP allocation in [7]. It can 658 also be noticed that AP-BPL is still in the leading position 659 among all schemes in terms of both throughput and delay. 660 Fig. 4 shows that at $P_{\text{max}}/N_0 = 5$ dB and $\lambda_p = 0.1$, AP-BPL 661

reduces the PU's average delay by up to 27% compared to AP-BSL, and 40% compared to EP-BPL. Moreover, we validate the obtained closed-form expressions for average PU delay via simulations. Theoretical and simulation results for AP-BSL perfectly coincide. However, for AP-BPL, the slight deviation between theory and simulations is attributed to the relaxation of the constraint $\mathbf{h}_{\rm I} < \mathbf{h}_{r^*}$.

Finally, we plot the average powers transmitted by the 669 SUs in Fig. 5, i.e., average P_{s^*} and P_{r^*} , normalized to N_0 , 670 versus P_{max}/N_0 . Clearly, the AP-based schemes consume 671 significantly less power than the EP assignment represented 672 by the 45° line. Power adaptation results approximately in 673 50% reduction in energy consumption at the SUs, compared to 674 equal power allocation, at $P_{\text{max}}/N_0 = 15$ dB. For the average 675 power transmitted on the link $s^* \rightarrow D_s$, the first intuition that 676 comes to mind is that AP-BSL policy results in the minimum 677 average power. However, this is only true at high $P_{\rm max}/N_0$ 678 values. It is noticed that the results corresponding to AP-BPL 679 show slightly less power consumption than that of AP-BSL 680 at low P_{max}/N_0 values. This behavior approximately holds 681 till $P_{\text{max}}/N_0 = 10$ dB. This is attributed to the nature of 682 the proposed AP policy which sets s^* silent if its maximum 683 power constraint is not sufficient to satisfy the condition 684 of success (5). Since in AP-BSL, s^* always sees the best 685 link to D_s , the number of slots in which it remains idle is 686 less than that in AP-BPL. This yields a higher throughput 687



Fig. 5. Average SUs' transmitted power normalized to N_0 versus P_{max}/N_0 .

at the expense of slightly higher average transmitted power. The same argument holds for comparing selection policies on the link $r^* \rightarrow D_p$.

691 A. Discussion on the Assumptions

The above system analysis is performed under the assump-692 tion of fully-backlogged SUs. The motivation behind this 693 assumption is two-fold. First, backlogged SUs represent the 694 worst case scenario from the PU's point of view. Since we 695 consider cooperative communications, a portion of the PU's 696 data is delivered to its intended destination via the relay 697 link, i.e., $r^* \rightarrow D_p$. However, the transmission of secondary 698 packets causes interference to the relay link as indicated 699 earlier. This interference is persistent in case of backlogged 700 SUs. Therefore, our results can be considered as a lower bound 701 on the achievable performance of the PU, i.e., a lower bound 702 on throughput and upper bound on delay. Furthermore, the 703 backlogged SUs assumption mitigates the interaction between 704 the queues of the SUs. This renders the system mathematically 705 tractable. Nevertheless, stochastic arrivals to the SUs' queues 706 can still be considered and queues interaction can be tackled 707 using the dominant system approach originally introduced 708 in [26]. However, this is out of the scope of the paper. 709

It is worth noting that in the derivations corresponding 710 to BPL-based schemes, i.e., in Sections VII and VII of 711 the Appendix, we consider \mathbf{h}_{I} and $\mathbf{h}_{r^{*}}$ independent random 712 variables. However, they are coupled through the constraint 713 $\mathbf{h}_{\rm I} < \mathbf{h}_{r^*}$. This constraint is an immediate consequence of 714 the BPL node selection policy. We relax this constraint to 715 render the problem mathematically tractable. Nevertheless, 716 we quantify the effect of relaxing this constraint on the 717 obtained closed-form expressions for f_{r^*} through numerical 718 simulation results presented in Section VI. 719

Finally, we assume that SUs perfectly sense the PU's activity. This assumption has been made to avoid adding further complexity to the analysis which might distort the main message behind the paper. Nevertheless, imperfect sensing has been studied extensively in the literature. Reference [27] presents a comprehensive survey of spectrum sensing techniques in cognitive radio networks.

VII. CONCLUSION

We discuss a power allocation policy for cognitive radio 728 networks with multiple relays and propose different relaying 729 protocols depending on the network utility function. The 730 effect of SU power adaptation on throughput and average 731 delay is thoroughly investigated. We derive the closed-form 732 expressions for the achieved throughput and average delay and 733 validate the results through numerical simulations. Dynami-734 cally adapting the transmission powers at the SUs according 735 to the channel conditions results in substantial improve-736 ment in primary and secondary throughput. The SUs under 737 EP-based schemes always transmit at maximum power. This 738 results in excessive interference on the relay link which is 739 not the case for the AP-based schemes. Power adaptation is 740 performed at the SUs to transmit with the minimum power 741 required for the successful transmission. To further benefit 742 the system, the SUs back-off if their maximum permissible 743 power is not sufficient to yield a successful transmission and 744 avoid guaranteed outage events. The back-off benefits the other 745 transmitting SU by reducing the incurred interference and 746 thereby, causes throughput increase. The AP-based schemes 747 are shown to reduce the average queuing delay encountered 748 by the PU packets compared to their EP-based counterparts. 749 We perform mathematical analysis of the proposed schemes 750 and show numerically that the AP-based schemes save energy; 751 and achieve higher throughput and lower delay simultaneously. 752

DISTRIBUTIONS OF
$$\mathbf{h}_{r^*}$$
, \mathbf{h}_{I} , and \mathbf{h}_{s^*} for BSL 75

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Referring to the policy described in Section IV-B.1,

$$\mathbf{h}_{s^*} = \max_{i \in \{1, \dots, N\}} \mathbf{h}_{s_i}.$$
 (44) 75

Therefore, the probability density function (PDF) of \mathbf{h}_{s^*} is

$$\mathcal{P}_{\mathbf{h}_{s^*}}(h) = Ne^{-h}(1-e^{-h})^{N-1}, \quad h \ge 0.$$
 (45) 758

As indicated earlier, the fact that s^* has the best link to D_s ⁷⁵⁹ gives absolutely no information about its link quality to D_p ⁷⁶⁰ and hence, ⁷⁶¹

$$\mathcal{P}_{\mathbf{h}_{\mathrm{I}}}(h) = e^{-h}, \quad h \ge 0.$$
 (46) 762

On the other hand,

$$\mathcal{P}_{\mathbf{h}_{r^*}}(h) = (N-1)e^{-h}(1-e^{-h})^{N-2}, \quad h \ge 0.$$
 (47) 764

We present a rigorous argument to prove that (47) 765 is true. Consider the 2N random variables represent-766 ing the link qualities of the N SUs to D_p and D_s . 767 The SU with the best link to D_s is selected to transmit a 768 packet of its own. This leaves (N-1) possible candidates for 769 relaying a primary packet to D_p . Among the (N-1) random 770 variables representing the link qualities of these candidates to 771 D_p , their maximum is selected. This maximum has one of the 772 following two possibilites. 773

• It is the second maximum of $\{\mathbf{h}_{r_i}\}_{i=1}^N$. This occurs only when the same SU has the best link to both D_p and D_s simultaneously. A specific SU has the best link to both destinations simultaneously with probability $1/N^2$. 777

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• It is the maximum of $\{\mathbf{h}_{r_i}\}_{i=1}^N$. This occurs whenever s^* is not the SU having the best link to D_p , which has a probability 1 - (1/N).

The average distribution corresponding to the two possibilities presented above with their respective probabilities is exactly the same as the distribution of a maximum of (N - 1) i.i.d. exponential random variables with means 1 each. This is an easy-to-show fact using order statistics arguments, omitted for brevity. The proof of (47) is then concluded.

790 APPENDIX B 791 DISTRIBUTIONS OF \mathbf{h}_{r^*} , \mathbf{h}_{I} , and \mathbf{h}_{s^*} for BPL

According to the policy described in Section IV-B2,

$$\mathbf{h}_{r^*} = \max_{i \in \{1, \dots, N\}} \mathbf{h}_{r_i}.$$
 (48)

(49)

(50)

Therefore, the PDF of \mathbf{h}_{r^*} is

95
$$\mathscr{P}_{\mathbf{h}_{r^*}}(h) = Ne^{-h}(1-e^{-h})^{N-1}, \quad h \ge 0.$$

796 On the other hand,

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$$\mathcal{P}_{\mathbf{h}_{e^*}}(h) = (N-1)e^{-h}(1-e^{-h})^{N-2}, \quad h \ge 0.$$

⁷⁹⁸ An argument similar to that used to derive the distribution of ⁷⁹⁹ \mathbf{h}_{r^*} in Appendix A is used to derive (50).

The SU with the best link to D_p is selected to relay a 800 primary packet. This eliminates the possibility that s^* has the 801 best link to D_p , i.e., \mathbf{h}_{I} can not be the maximum of $\{\mathbf{h}_{r_i}\}_{i=1}^N$. 802 In other words, \mathbf{h}_{I} can possibly be the *k*th order statistic of the N random variables $\{\mathbf{h}_{r_{i}}\}_{i=1}^{N}$, where k = 1, ..., N - 1. The 803 804 kth order statistic is by convention the kth smallest random 805 variable. It remains to note that after the selection of r^* , 806 the remaining (N-1) SUs possess equal probabilities of 807 having the best link to D_s . Consequently, $\mathbf{h}_{\rm I}$ is equally likely 808 to be any *k*th order statistic of $\{\mathbf{h}_{r_i}\}_{i=1}^N$, k = 1, ..., N - 1. Then, the average distribution of these order statistics is 809 810 given by 811

$$\mathfrak{P}_{\mathbf{h}_{\mathrm{I}}}(h) = \frac{N}{N-1} \sum_{k=1}^{N-1} \binom{N-1}{k-1} e^{-h(N-k+1)} \times (1-e^{-h})^{k-1}, \quad h \ge 0.$$
(51)

814 APPENDIX C 815 DERIVATION OF f_{r^*} AND f_{s^*} FOR EP-BSL

Using (1) and (4) along with the description of power allocation and node selection policies provided in Sections IV-A1 and IV-B1, respectively, we have

$$f_{r^*} = \mathbb{P}\left[\mathbf{h}_{r^*} > a + \frac{\mathbf{h}_{\mathrm{I}}}{b}\right].$$
 (52)

820 Then, total probability theory implies that

$$f_{r^*} = \int_0^\infty \mathbb{P}\left[\mathbf{h}_{r^*} > a + \frac{h}{b}\right] \mathcal{P}_{\mathbf{h}_{\mathbf{I}}}(h) dh$$
(53)

Thus, (53) is readily solved via substituting by the distributions of the random variables \mathbf{h}_{I} and \mathbf{h}_{r^*} provided in (46) and (47), respectively. We first note that

$$\mathbb{P}\left[\mathbf{h}_{r^*} > w\right] = 1 - (1 - e^{-w})^{N-1}, \quad w \ge 0$$
 (54) 825

and then use (54) with $w = a + \frac{h}{b}$ in (53) to get

$$f_{r^*} = \int_0^\infty \left[1 - \left[1 - e^{-\left(a + \frac{h}{b}\right)} \right]^{N-1} \right] .e^{-h} dh.$$
 (55) 827

To solve this integration, we use the binomial theorem

$$\left[1 - e^{-\left(a + \frac{h}{b}\right)}\right]^{N-1} = \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k e^{-k\left(a + \frac{h}{b}\right)}.$$
 (56) 825

We substitute by (56) in (55). Then, the integral solution renders f_{r^*} as in (29).

At the SUs side, we depend on (1) and (3) to write

$$f_{s^*} = \mathbb{P}\left[\mathbf{h}_{s^*} > a\right] = 1 - \beta^N \tag{57}$$
⁸³³

which follows directly from (45). This verifies f_{s^*} in (30).

APPENDIX D 835

DERIVATION OF f_{r^*} AND f_{s^*} FOR EP-BPL 836

We use the description of power allocation and node selection policies presented in Sections IV-A1 and IV-B2, respectively. Using (1) and (4), f_{r^*} is given by (52) which is the same as (53) through total probability theory. The distributions of \mathbf{h}_{r^*} and \mathbf{h}_{I} given by (49) and (51), respectively, are used to solve the integral in (53) using similar steps to that presented in Appendix C. This renders f_{r^*} as given in (31).

An SU transmits on the best link to D_s only when Q_r is 844 empty. Therefore, 845

$$f_{s^*} = \mathbb{P}\left[\left.\bar{\mathcal{O}}_{s^*}\right|\mathsf{B}\right]\mathbb{P}\left[\mathsf{B}\right] + \mathbb{P}\left[\left.\bar{\mathcal{O}}_{s^*}\right|\bar{\mathsf{B}}\right]\mathbb{P}\left[\bar{\mathsf{B}}\right]$$
(58) 846

where O_{S^*} denotes the outage event on the secondary link, and B denotes the event that Q_r is non-empty. A bar over an event's symbol denotes its complement. Little's theorem [22] implies that

$$\mathbb{P}\left[\mathsf{B}\right] = \gamma \tag{59} \quad \mathbf{85}$$

where γ is given by (35). In (59), we use the arrival and service rates of Q_r presented on both sides of (13), respectively. Next, we compute the probability of packet success on the secondary link when Q_r is busy. From (1) and (3), we have

$$\mathbb{P}\left[\left.\bar{O}_{s^*}\right|\mathsf{B}\right] = \mathbb{P}\left[\left.\mathbf{h}_{s^*} > a\right|\mathsf{B}\right] = 1 - \beta^{N-1}. \tag{60}$$

This follows from the distribution of \mathbf{h}_{s^*} given by (50). On the other hand, if Q_r is empty, s^* transmits on the best link among $\mathbb{S} \to D_s$, i.e., $\mathbf{h}_{s^*} = \max_{i \in \{1, \dots, N\}} h_{s_i}$. Thus, we have 859

$$\mathbb{P}\left[\left.\bar{O}_{s^*}\right|\bar{\mathsf{B}}\right] = \mathbb{P}\left[\left.\mathbf{h}_{s^*} > a\right|\bar{\mathsf{B}}\right] = 1 - \beta^N. \tag{61}$$

We substitute by the results of (59), (60), and (61) in (58). ⁸⁶¹ This verifies that f_{s^*} is given by (34). ⁸⁶²

863APPENDIX E864DERIVATION OF f_{r^*} AND f_{s^*} FOR AP-BSL

Using total probability theory, we write

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$$f_{r^*} = \mathbb{P}\left[\left.\bar{O}_{r^*}\right|O_{s^*}\right]\mathbb{P}\left[O_{s^*}\right] + \mathbb{P}\left[\left.\bar{O}_{r^*}\right|\bar{O}_{s^*}\right]\mathbb{P}\left[\bar{O}_{s^*}\right]$$
(62)

where O_{r^*} denotes the outage event on the relay link. In (62), we take into account the fact that s^* remains silent if P_{max} is not sufficient to satisfy (5). Therefore, we compute the probability of a successful transmission on the relay link in both cases of s^* activity, i.e., either active or silent. Thus, from (5), we have

$$\mathbb{P}[O_{s^*}] = \mathbb{P}[\mathbf{h}_{s^*} < a] = \beta^N.$$
(63)

This can directly be verified using the distribution of \mathbf{h}_{s^*} presented in (45). In the event of a sure outage on the secondary link, s^* refrains from transmission. We then plug $P_{s^*} = 0$ into (6) and write

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$$\mathbb{P}\left[\left.\bar{O}_{r^*}\right|O_{s^*}\right] = \mathbf{P}[\mathbf{h}_{r^*} > a] = 1 - \beta^N. \tag{64}$$

This result is explained as follows. When s^* is silent, r^* is selected to be the SU with the best link to D_p to enhance the PU throughput. Thus, in this specific case, \mathbf{h}_{r^*} is the maximum of N exponential random variables with means 1 each. This renders $\mathbb{P}[\mathbf{h}_{r^*} > a] = 1 - \beta^N$.

On the other hand, when s^* is active, i.e., $\mathbf{h}_{s^*} \ge a$, we choose P_{s^*} to be the value that meets (5) with equality and plug it into (6). After some algebraic manipulation, we write

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$$\mathbb{P}\left[\left.\bar{O}_{r^*}\right|\left.\bar{O}_{s^*}\right] = \mathbb{P}\left[\mathbf{h}_{\mathrm{I}} \leq b\left(\frac{\mathbf{h}_{r^*}}{a} - 1\right)\mathbf{h}_{s^*}\right|\mathbf{h}_{s^*} \geq a\right].$$
 (65)

The first step towards solving (65) requires the computation of $\mathbb{P}[\mathbf{h}_{I} \leq z\mathbf{h}_{s^*} | \mathbf{h}_{s^*} \geq a]$ for an arbitrary $z \geq 0$. Proceeding with that, we have

$$\mathbb{P}[\mathbf{h}_{\mathrm{I}} \leq z\mathbf{h}_{s^*} | \mathbf{h}_{s^*} \geq a] = \frac{\mathbb{P}[\mathbf{h}_{\mathrm{I}} \leq z\mathbf{h}_{s^*}, \mathbf{h}_{s^*} \geq a]}{\mathbb{P}[\mathbf{h}_{s^*} \geq a]}.$$
 (66)

⁸⁹² The numerator of (66) can be computed as follows.

$$\mathbb{P}[\mathbf{h}_{\mathrm{I}} \leq z\mathbf{h}_{s^*}, \mathbf{h}_{s^*} \geq a] = \int_a^\infty \int_0^{zy} \mathcal{P}_{\mathbf{h}_{\mathrm{I}}}(x) \mathcal{P}_{\mathbf{h}_{s^*}}(y) dx dy \quad (67)$$

The distributions of \mathbf{h}_{I} and $\mathbf{h}_{s^{*}}$ are given by (46) and (45), respectively, and we use the fact that \mathbf{h}_{I} and $\mathbf{h}_{s^{*}}$ are independent. This information, along with the binomial theorem, is used to solve the double integral in (67). Thus,

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$$\mathbb{P}[\mathbf{h}_{\mathrm{I}} \le z\mathbf{h}_{s^*}, \mathbf{h}_{s^*} \ge a] = N \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k e^{-a(k+1)} \times \left[\frac{1}{k+1} - \frac{e^{-az}}{z+k+1}\right]. \quad (68)$$

⁹⁰⁰ Furthermore, we know from (63) that

$$\mathbb{P}\left[\bar{O}_{s^*}\right] = \mathbb{P}[\mathbf{h}_{s^*} \ge a] = 1 - \beta^N.$$
(69)

Then, we substitute by (68) and (69) in (66). Next, we use total probability theory to write (65) as

$$\mathbb{P}\left[\left.\bar{O}_{r^*}\right|\bar{O}_{s^*}\right]$$

$$= \int_a^{\infty} \mathbb{P}\left[\mathbf{h}_{\mathrm{I}} \le b\left(\frac{w}{a}-1\right)\mathbf{h}_{s^*}\right|\mathbf{h}_{s^*} \ge a\right] \mathcal{P}_{\mathbf{h}_{r^*}}(w)dw \quad (70)$$

where $\mathcal{P}_{\mathbf{h}_{r^*}}(.)$ is given by (47). We then substitute by the result of (66), with $z = b\left(\frac{w}{a} - 1\right)$, in (70). The solution of the not integral yields

$$\mathbb{P}\left[\left.\bar{O}_{r^*}\right|\left.\bar{O}_{s^*}\right]\right.$$
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$$= \frac{N}{\left(1-\beta^{N}\right)} \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^{k} e^{-a(k+1)} \left[I_{3}-I_{4}\right]$$
(71) 910

where I_3 and I_4 are given by (37) and (38), respectively. The derivation of (38) depends on the fact that 912

$$\int_{a}^{\infty} \frac{e^{-tw}}{w+c} dw = e^{tc} E_1[t(a+c)]$$
(72) 913

for any constants t and c. Substituting by (37) and (38) ⁹¹⁴ in (71), and using (63), (64), (69), and (71) in (62), f_{r^*} is ⁹¹⁵ shown to be given by (36). ⁹¹⁶

For the SUs, f_{s^*} is shown to be given by (57) following the same proof provided for the case of EP-BSL in Appendix C. 918

DERIVATION OF f_{r^*} AND f_{s^*} FOR AP-BPL

The derivation of f_{r^*} for AP-BPL follows the same footsteps of the derivation presented in Appendix E. However, the difference in the node selection policies induces different distributions for the random variables of interest. We can write f_{r^*} as in (62). First, we derive the first term in the RHS of (62) as follows.

$$\mathbb{P}[\mathcal{O}_{s^*}] = \mathbb{P}[\mathbf{h}_{s^*} < a] = \beta^{N-1}.$$
(73) 92

This follows from the distribution of \mathbf{h}_{s^*} presented in (50). 928 When s^* is silent, we plug $P_{s^*} = 0$ into (6) and write 929

$$\mathbb{P}\left[\left.\bar{O}_{r^*}\right|O_{s^*}\right] = \mathbb{P}[\mathbf{h}_{r^*} > a] = 1 - \beta^N \tag{74}$$

where the distribution of \mathbf{h}_{r^*} is given by (49). Then, we shift 931 our attention to the second term in the RHS of (62). When 932 s^* is active, i.e., $\mathbf{h}_{s^*} \ge a$, we choose P_{s^*} to be the value that 933 meets (5) with equality and plug it into (6). Then, we compute 934 the probability of success on the relay link given that s^* is 935 active as in (65). We solve (67) using the distributions of \mathbf{h}_{s^*} 936 and h_I in (50) and (51), respectively, along with the fact that 937 they are independent to get 938

$$\mathbb{P}\left[\mathbf{h}_{I} \leq z\mathbf{h}_{s^{*}}, \mathbf{h}_{s^{*}} \geq a\right] = \sum_{k=1}^{N-1} \sum_{\ell=0}^{k-1} \sum_{m=0}^{N-2} \binom{N-1}{k-1} \binom{k-1}{\ell} \binom{N-2}{m} \times \frac{N(-1)^{m+\ell}}{(N-k+\ell+1)} + \binom{N-2}{m} \times \frac{N(-1)^{m+\ell}}{(N-k+\ell+1)} + \binom{e^{-a(m+1)}}{(m+1)} - \frac{e^{-a(m+2(N-k+\ell+1)+1)}}{(m+2(N-k+\ell+1)+1)} \right]$$
(75) 94

for $z \ge 0$. Next, we substitute by the result of (75), with $z = b\left(\frac{w}{a} - 1\right)$, in (70) and solve the integral. After some algebraic manipulation, omitted for brevity, the second term in the right hand side of (62) is found to be equal to 945

$$\sum_{k=1}^{N-1} \sum_{\ell=0}^{k-1} \sum_{m=0}^{N-2} \binom{N-1}{k-1} \binom{k-1}{\ell} \binom{N-2}{m}$$
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$$\times \frac{(-1)^{m+\ell} N^2}{(N-k+\ell+1)} [I_5 - I_6]$$
(76) 947

where 948

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¹⁹ I₅ =
$$\frac{e^{-a(m+1)}}{(m+1)} \int_{a}^{\infty} e^{-w} (1-e^{-w})^{N-1} dw$$
 (77)

$$I_{6} = \sum_{n=0}^{N-1} \frac{a(-1)^{n} e^{-a(m+n+2-t)}}{(t-n-1)} \int_{a}^{\infty} \frac{e^{-tw}}{w+c} dw$$
(78)

and the terms t and c are given by (42) and (43), respectively. 951 The solution of the integral in (77) proves that I₅ is given 952 by (40). We use (72) to show that I_6 is given by (41). 953 Then, (73), (74), and (76) shows that f_{r^*} is given by (39). 954

On the other hand, f_{s^*} is shown to be given by (34) 955 following the same proof provided for the case of EP-BPL 956 in Appendix D. 957

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Energy-Aware Cooperative Wireless Networks With Multiple Cognitive Users

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Abstract—In this paper, we study and analyze cooperative 1 cognitive radio networks with arbitrary number of secondary 2 users (SUs). Each SU is considered a prospective relay for the 3 primary user (PU) besides having its own data transmission 4 demand. We consider a multi-packet transmission framework 5 that allows multiple SUs to transmit simultaneously because of 6 dirty-paper coding. We propose power allocation and scheduling 7 policies that optimize the throughput for both PU and SU with 8 minimum energy expenditure. The performance of the system 9 is evaluated in terms of throughput and delay under different 10 opportunistic relay selection policies. Toward this objective, 11 we present a mathematical framework for deriving stability con-12 ditions for all queues in the system. Consequently, the throughput 13 of both primary and secondary links is quantified. Furthermore, 14 a moment generating function approach is employed to derive a 15 closed-form expression for the average delay encountered by the 16 PU packets. Results reveal that we achieve better performance in 17 terms of throughput and delay at lower energy cost as compared 18 with equal power allocation schemes proposed earlier in the 19 literature. Extensive simulations are conducted to validate our 20 21 theoretical findings.

Index Terms—Cognitive relaying, opportunistic communica tion, throughput, delay, relay selection.

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I. INTRODUCTION

OGNITIVE radio networks have emerged as an efficient solution to the problem of spectrum scarcity and its under-utilization. In a cognitive radio network, the secondary

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users (SUs) exploit primary users' (PUs) period of inactivity 28 to enhance their performance provided that PUs' performance 29 remains unaffected. Depending on the mode of interaction 30 of the primary and the secondary users, the cognitive radio 31 networks are classified as interweave, underlay and overlay 32 networks. In the last decade or so, the industry and academia 33 has shown overwhelming interest in the application of cogni-34 tive radios in different networking solutions. Reference [2] 35 provides a comprehensive overview of the cognitive radio 36 fundamentals and research activities. 37

On the other hand, cooperative diversity has been widely 38 investigated in pursuit of combating multipath fading [3], [4]. 39 Incorporating cooperation into cognitive radio networks results 40 in substantial performance gains in terms of throughput and 41 delay for both primary and secondary nodes [5]. The SUs 42 help the PUs to transmit their data, and create opportu-43 nities for their own data transmission at the same time. 44 The cooperation between the PUs and the SUs vary from 45 just sharing information about queue states, channel state 46 information (CSI), and primary packet transmission activity 47 to the use of SUs as cognitive relays. Typically, relaying is 48 carried out over orthogonal channels due to the half-duplex 49 communication constraint at the relays [3]. However, some of 50 the recent solutions overcome this limitation by accommodat-51 ing simultaneous transmissions in a single slot [6]–[8]. This 52 is achieved through space-time coding [6] or dirty-paper 53 coding (DPC) [7], [8]. Conventionally, zero forcing and more 54 recently prior zero forcing [9] has been employed to mitigate 55 the SU signal interference with the PU signals. On the other 56 side, for cooperative cognitive radio networks with multiple 57 SUs with their own data transmission demands, employing 58 DPC allows one SU to transmit new data while the other SU 59 helps the PU by relaying its data. Thus, the spectral efficiency 60 of the system is enhanced. 61

In literature, there is a rich volume of recent work focusing 62 on cooperation in cognitive relay networks. The benefits 63 of cooperative relaying has been discussed and analyzed 64 in [10]–[12]. In [10], authors derive the maximum sustained 65 throughput of a single SU to maintain a fixed throughput 66 for PU with and without relaying. They used a dominant 67 system approach to guarantee the queue stability of both SU 68 and PU while overcoming the queues interaction. A cognitive 69 system comprising a single PU and multiple SUs along with 70 multiple relays is considered in [12], where a proportion of 71 the secondary relays help the PU in communication while a 72 relay selection is performed from the remaining relays to give 73 simultaneous access to the SU. The authors show that there 74

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exists an optimal number of cooperating relays with the PU 75 that achieve optimal outage performance. In [13], the authors 76 also discuss a cognitive relay selection problem using optimal 77 stopping theory. Reference [14] addresses a cognitive radio 78 cooperation model where the SU can transmit its data along 79 with primary transmission, but cooperates by deferring its 80 transmission when the PU is transmitting. The authors in [15] 81 address a cooperative cognitive relay network where both 82 primary and secondary nodes use cognitive relays for data 83 transmission. The relays help the PUs empty their queues 84 fast and thereby, the throughput for the SUs increases as a 85 result. SU throughput stability regions for cooperative cogni-86 tive networks have been derived for cooperative cognitive radio 87 networks in different settings in [9] and [15]. Reference [17] 88 investigates the energy efficiency in cognitive radio networks 89 via developing low-complexity algorithms for solving a joint 90 optimization problem of the spectrum sensing duration and the 91 transmit power of the cognitive users. 92

Krikidis et al. address different protocols for a cognitive 93 cooperative network and the stable throughput for both pri-94 mary and the secondary networks is derived. In this paper, 95 we adopt the model presented in [7] and employ DPC. 96 We consider a cognitive network with arbitrary number of 97 SUs co-existing with a PU and sharing one common relay 98 queue. We propose power allocation and scheduling poli-99 cies that enhance the throughput of both primary and sec-100 ondary links using the least possible energy expenditure. 101 The summary of the main contributions of this work is as 102 follows. 103

- We propose an energy-efficient adaptive power (AP) allocation scheme for the SUs that enhances the throughput of both primary and secondary links. Energy-efficient transmission is achieved via exploiting instantaneous CSI to adapt the transmission powers at all SUs.
- We introduce two SU scheduling policies, which pri oritize primary or secondary throughput enhancement
 according to the network requirements. We analyze the
 performance of both policies in conjunction with equal
 and adaptive power allocation schemes.

We develop a generic mathematical framework to derive 114 closed-form expressions for both PU and SU throughput, 115 and PU average delay. The mathematical analysis is 116 performed for an arbitrary number of SUs coexisting with 117 a PU. A detailed analysis is performed for each combi-118 nation of power allocation and SU scheduling policies. 119 We validate our theoretical findings via simulations. 120 Results reveal that AP-based schemes yield superior 121 performance compared to EP allocation proposed in [7], 122 with significantly less energy cost. 123

The rest of this paper is organized as follows. Section II 124 presents the information-theoretic background and preliminar-125 ies needed in the sequel. Section III introduces the system 126 model and the proposed cooperation strategy. The opportunis-127 tic relay selection and power allocation strategies are presented 128 in Section IV along with their mathematical analysis in 129 Section V. Numerical results are then presented in Section VI. 130 Finally, concluding remarks are drawn in Section VII. 131

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Fig. 1. Cognitive radio network model under consideration. The (logical) CSB is shown to coordinate the activities of the common relay queue.

II. BACKGROUND AND PRELIMINARIES

A. Dirty-Paper Coding

DPC was first introduced in [18] and we briefly state its 134 implication. Consider a channel with output $\mathbf{y} = \mathbf{x} + \mathbf{q} + \mathbf{z}$, 135 where \mathbf{x} , \mathbf{q} and \mathbf{z} denote the input, interference, and noise, 136 respectively. The input $\mathbf{x} \in \mathbb{C}^m$ satisfies the power con-137 straint $(1/m) \sum_{i=1}^{m} |x_i|^2 \le P_0$. We assume that **q** and **z** are 138 zero-mean Gaussian vectors with covariance matrices QI_m 139 and $N_0 \mathbf{I}_m$, respectively, where \mathbf{I}_m denotes the $m \times m$ identity 140 matrix. If the interference \mathbf{q} is unknown to both transmitter and 141 receiver, the channel capacity is given by $\log(1+P_0/(Q+N_0))$ 142 (bits/channel use). However, if \mathbf{q} is known to the transmitter 143 but not the receiver, the channel capacity is shown to be 144 the same as that of a standard "interference free" Gaussian 145 channel with signal-to-noise ratio P_0/N_0 using DPC. In other 146 words, if the interference is known a priori at the transmitter, 147 DPC renders the link between the transmitter and its intended 148 receiver interference-free. 149

B. Channel Outage

We present the basic definition of an outage event and 151 the corresponding outage probability calculation. Consider a 152 channel with output $\mathbf{y} = \sqrt{\mathbf{h}}\mathbf{x} + \mathbf{z}$, where $\sqrt{\mathbf{h}}$ and \mathbf{x} denote 153 the fading coefficient and the input, respectively. Moreover, 154 the noise z is modelled as zero-mean circularly symmetric 155 complex Gaussian random variable with variance N_0 . For a 156 target transmission rate R_0 , an outage occurs if the mutual 157 information between the input and output is not sufficient to 158 support that rate. The probability of such event, for a channel 159 with average power constraint P_0 , is 160

$$\mathbb{P}\left[\mathbf{h} < \frac{2^{R_0} - 1}{P_0/N_0}\right].$$
 (1) 16

III. SYSTEM MODEL 162

We consider the cognitive radio system shown in Fig. 1. 163 The system comprises a PU p that transmits its packets to a 164 primary destination D_p . A cognitive network consisting of an 165 arbitrary number of SUs coexists with the primary network. 166 The number of SUs is denoted by N and we refer to the set of 167 SUs by $\mathbb{S} = \{s_i\}_{i=1}^N$. Each SU has its own data that requires 168 to be delivered to a common secondary destination D_s . All 169 nodes are equipped with infinite capacity buffers. Time is 170

slotted, and the transmission of a packet takes exactly one time 171 slot. The duration of a time slot is normalized to unity and 172 hence, the terms power and energy are used interchangeably 173 in the sequel. We take into account the bursty nature of the 174 source through modelling the arrivals at the PU as a Bernoulli 175 process with rate λ_p (packets/slot). In other words, at any 176 given time slot, a packet arrives at the PU with probability 177 $\lambda_p < 1$. The arrival process at the PU is independent and 178 identically distributed (i.i.d.) across time slots. On the other 179 hand, the SUs are assumed backlogged, i.e., SUs always 180 have packets awaiting transmission. We assume that the SUs 181 perfectly sense the PU's activity, i.e., there is no chance of 182 collision between the PU and any of the secondary users. 183 A node that successfully receives a packet broadcasts an 184 acknowledgment (ACK) declaring the successful reception 185 of that packet. ACKs sent by the destinations are assumed 186 instantaneous and heard by all nodes error-free. 187

The channel between every transmitter-receiver pair exhibits 188 frequency-flat Rayleigh block fading, i.e., the channel coeffi-189 cient remains constant for one time slot and changes indepen-190 dently from one slot to another. The scalars $\mathbf{h}_{r_i}[n]$ and $\mathbf{h}_{s_i}[n]$ 191 denote the absolute squared fading coefficient of the channels 192 that connect the *i*th SU to D_p and D_s , respectively, at the *n*th 193 time slot. Similarly, the absolute squared fading coefficient of 194 the channels that connect the PU to D_p and s_i , at the *n*th time 195 slot, are denoted by $\mathbf{h}_p[n]$ and $\mathbf{h}_{ps_i}[n]$, respectively. According 196 to the Rayleigh fading assumption, $\mathbf{h}_{r_i}[n]$, $\mathbf{h}_{s_i}[n]$, and $\mathbf{h}_{ps_i}[n]$ 197 are exponential random variables with means σ^2 , for all 198 $i = 1, \ldots, N$. We denote an exponential random variable 199 with mean σ^2 by $\exp(\sigma^2)$. Then, we have $\mathbf{h}_p[n] \sim \exp(\sigma_p^2)$. 200 All links are considered statistically equivalent except for the 201 link $p \to D_p$. We assume that $\sigma_p^2 < \sigma^2$ to demonstrate the 202 benefits of cooperation [19]. For the ease of exposition, we set 203 $\sigma^2 = 1$ throughout the paper. All communications are subject 204 to additive white Gaussian noise of variance N_0 . 205

Next, we present the queuing model of the system followed 206 by the description of the employed cooperation strategy. 207

A. Queuing Model 208

The queues involved in the system analysis, shown in Fig. 1, 209 are described as follows: 210

- Q_p : a queue that stores the packets of the PU correspond-211 ing to the external Bernoulli arrival process with rate λ_p . 212
- Q_{s_i} : a queue that stores the packets at the *i*th SU, where 213 $i \in \{1,\ldots,N\}.$ 214

• Q_r : a queue that stores PU packets to be relayed to D_p . 215 Having independent relay queues for all SUs makes exact 216 performance analysis intractable with the increasing number 217 of users. To address this complexity, Krikidis et al. introduced 218 the idea of a common 'fictitious' relay queue Q_r in [7], which 219 is maintained by a so-called cluster supervision block (CSB) 220 that controls and synchronizes all the activities of the cognitive 221 cluster. Along the lines of [7], we assume the existence 222 of a common relay such that SUs can perfectly exchange 223 information with the CSB with a negligible overhead. The 224 channels $\mathbb{S} \to D_p, D_s$ are assumed known instantaneously at 225 the CSB [7], [20]. 226

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The instantaneous evolution of queue lengths is captured as 227

$$\mathbf{Q}_{i}[n+1] = (\mathbf{Q}_{i}[n] - \mathbf{L}_{i}[n])^{+} + \mathbf{A}_{i}[n], \quad i \in \{p, r\} \cup \mathbb{S} \quad (2) \quad 228$$

where $(x)^+ = \max(x, 0)$ and $\mathbf{Q}_i[n]$ denotes the number of 229 packets in the *i*th queue at the beginning of the *n*th time slot. 230 The binary random variables taking values either 0 or 1, $L_i[n]$ 231 and $A_i[n]$, denote the departures and arrivals corresponding to 232 the *i*th queue in the *n*th time slot, respectively. 233

B. Cooperation Strategy

The employed cooperative scheme is described as follows.

- 1) The PU transmits a packet whenever Q_p is non-empty. 236
- 2) If the packet is successfully decoded by D_p , it broadcasts an ACK and the packet is dropped from Q_p .
- 3) If the packet is not successfully received by D_p yet successfully decoded by at least one SU, an ACK is broadcasted and the packet is buffered in Q_r and dropped from Q_p .
- 4) If D_p and S fail to decode the packet, it is kept at Q_p for retransmission in the next time slot.
- 5) When the PU is sensed idle, if Q_r is non-empty, two out 245 of all SUs transmit simultaneously. One SU is selected 246 to relay a packet from Q_r to D_p and is denoted by r^* . 247 Another SU is selected to transmit a packet of its own 248 to D_s and is denoted by s^* . Otherwise, if Q_r is empty, 249 one SU is selected to transmit a packet to D_s .¹ The SUs' selection policies are explained in Section IV-B.
- If the packets transmitted by the SUs are successfully 6) 252 received by their respective destinations, ACKs are 253 broadcasted and these packets exit the system. Other-254 wise, the packet that experiences unsuccessful transmis-255 sion is kept at its queue for later retransmission. 256

IV. POWER ALLOCATION AND NODE SELECTION

In this section, we introduce the adaptive power allocation 258 and opportunistic relay selection strategies for an arbitrary 259 number of SUs, N > 2. We propose a power allocation policy 260 that minimizes energy consumption at each SU as compared 261 to a fixed power allocation policy in [7]. In the sequel, node 262 selection policy refers to the choice of the SU that relays a pri-263 mary packet from Q_r to D_p , and the SU that transmits a packet 264 from its own queue to D_s , i.e., the selection of r^* and s^* . 265 The availability of CSI for all the channels (and thereby 266 incurred interference) at the CSB is exploited to perform power 267 allocation and node selection online, i.e., every time slot. 268

A. Power Allocation

Whenever Q_p is non-empty, the PU transmits a packet 270 with average power P_0 . However, when the PU is idle and 271 Q_r is non-empty, two SUs out of N transmit simultane-272 ously by employing DPC [18]. One SU relays a primary 273 packet to D_p while the other transmits a secondary packet 274 to D_s . Since all SUs can perfectly exchange information with 275

¹Note that two SUs can be selected for transmission if Q_r is empty. However, this requires multi-packet reception capability at the secondary destination which is out of the scope of this paper.

the CSB, Q_r is accessible by both SUs selected for transmis-276 sion. Therefore, the transmission of r^* is considered a priori 277 known interference at s^* . Accordingly, s^* adapts its signal 278 to see an interference-free link to D_s using the result stated 279 in Section II-A. On the other hand, s^* transmits a packet 280 from its own queue which is not accessible by r^* . Thus, 281 the transmission of s^* causes an interference on the relay 282 link, i.e., $r^* \rightarrow D_p$. The achievable rate region on this 283 Z-interference channel at the *n*th time slot is given by 284

$$\mathbf{R}_{s*}[n] = \log\left[1 + \frac{P_{s*}[n]\mathbf{h}_{s*}[n]}{N_0}\right]$$
(3)

$$\mathbf{R}_{r^*}[n] = \log\left[1 + \frac{P_{r^*}[n]\mathbf{h}_{r^*}[n]}{N_0 + P_{s^*}[n]\mathbf{h}_{\mathbf{I}}[n]}\right]$$

where $P_{s*}[n]$ and $P_{r*}[n]$ denote the instantaneous trans-287 mit powers of s^* and r^* , respectively. The instantaneous 288 absolute squared fading coefficients of the secondary, relay, 289 and interference links are denoted by $\mathbf{h}_{s^*}[n]$, $\mathbf{h}_{r^*}[n]$, and $\mathbf{h}_{I}[n]$, 290 respectively. We denote the links $s^* \rightarrow D_s$, $r^* \rightarrow D_p$, 291 and $s^* \rightarrow D_p$ by the secondary, the relay, and the interference 292 link, respectively. Hereafter, we omit the temporal index 293 *n* for simplicity. Nevertheless, it is implicitly understood that 294 power allocation and node selection are done on a slot-by-295 slot basis. In this work, we focus on developing an adaptive 296 power allocation scheme for the transmitting SUs that use 297 a fixed transmission rate R_0 . Specifically, our multi-criterion 298 objective is to enhance primary and secondary throughput 299 while minimizing the energy consumption at each SU. The 300 rates given by (3) and (4) stimulate thinking about how power 301 is allocated to both transmitting SUs. 302

Next, we investigate two different power allocation policies 303 for the SUs, namely, equal power (EP) allocation and adaptive 304 power (AP) allocation. It is worth noting that power allocation 305 and node selection are performed for the SUs since we have 306 no control on the PU. Thus, in the following lines, we focus 307 on the slots in which the PU is idle. 308

1) Equal Power Allocation: This policy assigns equal trans-309 mission powers to the SUs as proposed in [7] and serves as 310 a baseline scheme in this work. Whenever an SU transmits, 311 it uses an average power P_{max} . Specifically, if an SU is 312 transmitting alone, e.g., Q_r is empty, it uses a power P_{max} . 313 If two SUs transmit simultaneously, e.g., Q_r is non-empty, 314 $P_{s^*} = P_{r^*} = P_{\max}.$ 315

2) Adaptive Power Allocation: Unlike EP allocation, 316 we exploit the CSI available at the CSB to propose an AP 317 allocation scheme that minimizes the average power consump-318 tion at each SU. We use (3) and (4) along with (1) to derive 319 conditions on P_{s^*} and P_{r^*} for successful transmission at a 320 target transmission rate R_0 . These conditions are 321

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$$P_{s^*} \ge \frac{(2^{R_0} - 1)N_0}{\mathbf{h}_{s^*}} \tag{5}$$

$$P_{r^*} \ge \frac{(2^{R_0} - 1)}{2}$$

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$$= \frac{(2^{\kappa_0} - 1)[N_0 + P_{s^*} \mathbf{h}_{\mathrm{I}}]}{\mathbf{h}_{r^*}}.$$
 (6)

A transmitter that violates the condition on its transmis-324 sion power experiences a sure outage event. Furthermore, 325 we impose a maximum power constraint at each SU, where 326 $P_{s^*}, P_{r^*} \leq P_{\max}$. It is worth noting that P_{s^*} is computed first 327

according to (5) followed by the computation of P_{r^*} according 328 to (6). In a given slot, if P_{max} is less than the power required to 329 guarantee a successful transmission for a given SU, i.e., P_{max} 330 is less than the right hand sides of either (5) or (6), the CSB 331 sets the power of that SU to zero to avoid a guaranteed outage 332 event. Clearly, this results in increasing the throughput of the 333 PU due to reduction in the amount of interference caused 334 by the transmission of s^* on the relay link in the time slots 335 where s* refrains from transmitting. Moreover, compared to 336 EP allocation, energy wasted in slots where a sure outage event 337 occurs is now saved. 338

B. Node Selection Policies

(4)

We consider a system that assigns full priority to the 340 PU to transmit whenever it has packets. Therefore, the SUs 341 continuously monitor the PU's activity seeking an idle time 342 slot. When the PU is sensed idle, the SUs are allowed 343 to transmit their own and/or a packet from the common 344 queue Q_r . Note that it is possible to transmit only one packet 345 by the SUs in the following scenarios: 346

- 1) If Q_r is empty, i.e., no primary packet to be relayed. 347 Then, we select the SU with the best channel to D_s .
- 2) Q_r is non-empty, but r^* or s^* is set silent by the 349 CSB to avoid a guaranteed outage event on the $r^* \rightarrow$ 350 D_p or $s^* \to D_s$ link. Note that CSI for transmission is 351 assumed to be known at CSB and outage event (due to 352 power limitation) can be predicted before transmission 353 as discussed in Section IV-A.2. In this case, we choose 354 the transmitting SU as the one with the best instanta-355 neous link to the intended destination. For example, if r^* 356 is silent and s^* is transmitting alone, the SU with the 357 best link between $\mathbb{S} \to D_s$ transmits. 358

The case for the simultaneous transmission of two SUs is 359 the main topic for investigation in this paper. If the two 360 transmissions occur simultaneously, the transmitting SUs are 361 selected according to one of the following policies. 362

1) Best Secondary Link (BSL): In this policy, the utility 363 function to be maximized is the SU throughput. Therefore, 364 we choose the SU that transmits a packet of its own as the 365 one with the best instantaneous link to D_s , i.e., 366

$$\mathbf{h}_{s^*} = \max_{i \in \{1, \dots, N\}} \mathbf{h}_{s_i}.$$
 (7) 36

Among the remaining (N-1) SUs, the one with the best 368 instantaneous link to D_p is chosen to be r^* . 369

2) Best Primary Link (BPL): In this policy, unlike BSL, 370 the utility function to be maximized is PU throughput. Thus, 371 we choose the SU that relays a primary packet from Q_r as 372 the one with the best instantaneous link to D_p , i.e., 373

$$\mathbf{h}_{r^*} = \max_{i \in \{1, \dots, N\}} \mathbf{h}_{r_i}.$$
 (8) 374

Among the remaining (N-1) SUs, the one with the best 375 instantaneous link to D_s is chosen to be s^* . 376

It is worth noting that all links $\mathbb{S} \rightarrow D_p, D_s$ are sta-377 tistically independent. Thus, at any given time slot, if a 378 certain SU has the best instantaneous channel to a cer-379 tain destination, e.g., D_p , we can not infer any infor-380 mation about its link quality to the other destination, 381

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e.g., D_s . Hence, $\forall i \in \{1, ..., N\}$, s_i can have the best link to D_p/D_s irrespective of the quality of its link to the other destination.

So far, we have introduced two policies for each of the power allocation and SU scheduling policies. Thus, we have four different cases arising from the possible combinations of these policies. Next, we proceed with the performance analysis of the system for each case.

V. THROUGHPUT AND DELAY ANALYSIS

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In this section, we conduct a detailed analysis for the system 391 performance in terms of throughput and delay. Towards this 392 objective, we derive the stability conditions on the queues with 393 stochastic packet arrivals, namely, Q_p and Q_r . The stability 394 of a queue is loosely defined as having a bounded queue size, 395 i.e., the number of packets in the queue does not grow to infin-396 ity [19]. Furthermore, we analyze the average queuing delay 397 of the primary packets. We obtain a closed-form expression 398 for this delay through deriving the moment generating func-399 tion (MGF) of the joint lengths of Q_p and Q_r . It is worth not-400 ing that the SUs' queues are assumed backlogged and hence, 401 no queueing delay analysis is performed for the secondary 402 packets. In the following lines, we provide a general result for 403 the throughput of the primary and secondary links as well as 404 the delay of primary packets. Then, we proceed to highlight 405 the role of the proposed power allocation and node selection 406 policies. We first introduce some notation. The probabilities of 407 successful transmissions on the relay and secondary links are 408 denoted by f_{r^*} and f_{s^*} , respectively. A transmission on the 409 link $p \rightarrow D_p$ is successful with probability f_p . In addition, 410 the probability that at least one SU successfully decodes a 411 transmitted primary packet is denoted by f_{ps} . 412

Theorem 1: The maximum achievable PU throughput for
the system shown in Fig. 1, under any combination of power
allocation and node selection policies, is given by

416
$$\lambda_p < \frac{f_{r^*}[f_p + (1 - f_p)f_{ps}]}{f_{r^*} + (1 - f_p)f_{ps}}$$
(9)

417 while the throughput of the SU $s_i \in S$ is given by

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$$\mu_{s_i} = \frac{1}{N} \left[1 - \frac{\lambda_p}{f_p + (1 - f_p) f_{ps}} \right] f_{s^*}.$$
 (10)

⁴¹⁹ *Proof:* We use Loynes' theorem [21] to establish the ⁴²⁰ stability conditions for Q_p and Q_r . The theorem states that ⁴²¹ if the arrival and service processes of a queue are stationary, ⁴²² then the queue is stable if and only if the arrival rate is strictly ⁴²³ less than the service rate. Therefore, for Q_p to be stable, ⁴²⁴ the following condition must be satisfied

$$\lambda_p < \mu_p \tag{11}$$

where μ_p denotes the service rate of Q_p . A packet departs Q_p if it is successfully decoded by at least one node in $\mathbb{S} \cup \{D_p\}$. Thus, μ_p is given by

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$$\mu_p = f_p + (1 - f_p) f_{ps}.$$
 (12)

430 Similarly, Q_r is stable if

$$\frac{\lambda_p}{\mu_p}(1-f_p)f_{ps} < \left[1-\frac{\lambda_p}{\mu_p}\right]f_{r^*}.$$
(13)

A PU's packet arrives at Q_r if Q_p is non-empty and an 432 outage occurs on the direct link $p \rightarrow D_p$ yet no outage 433 occurs at least on one link between $p \rightarrow S$. From Little's 434 theorem [22], we know that probability of Q_p being non-435 empty equals λ_p/μ_p . This explains the rate of packet arrivals 436 at Q_r shown on the left hand side (LHS) of (13). The right 437 hand side (RHS) represents the service rate of Q_r . A packet 438 departs Q_r if Q_p is empty and there is no outage on the 439 link $r^* \to D_p$. Rearranging the terms of (13), we obtain the 440 maximum achievable PU throughput as given by (9) provided 441 that μ_p is given by (12). It is worth noting that (9) provides 442 a tighter bound on λ_p than (11) due to the multiplication of 443 μ_p in (9) by a term less than one. 444

On the other hand, we compute the throughput of SUs 445 by calculating the service rate of their queues since they 446 are assumed backlogged. Due to the symmetric configura-447 tion considered, i.e., statistically equivalent links $\mathbb{S} \to D_s$, 448 the throughput of all SUs is the same. For $s_i \in S$, a packet 449 departs Q_{s_i} if Q_p is empty, s_i is selected to transmit a packet 450 of its own and no outage occurs on the link $s_i \rightarrow D_s$. 451 Due to symmetry, at any time slot, all SUs have equal 452 probabilities to be selected to transmit a packet from their own 453 queues, i.e., $\mathbb{P}[s^* = s_i] = 1/N \ \forall i \in \{1, \dots, N\}$. Therefore, 454 the SUs' throughput is given by (10) provided that μ_p is 455 given by (12). \Box 456

Next, we develop a mathematical framework to analyze the average queuing delay for the PU's packets.

Theorem 2: The average queuing delay encountered by the PU packets in the system shown in Fig. 1, under any combination of power allocation and node selection policies, is

$$\tau = \frac{N_p + N_r}{\lambda_p} \tag{14}$$

where N_p and N_r , the average lengths of Q_p and Q_r , 463 respectively, are given by 464

$$N_p = \frac{-\lambda_p^2 + \lambda_p}{\mu_p - \lambda_p} \tag{15}$$

$$N_r = \frac{r\lambda_p^2 + s\lambda_p}{\delta\lambda_p^2 + \zeta\lambda_p + \eta} \tag{16}$$

and

$$r = f_{ps}(1 - f_p) \left[\frac{f_{r^*} - f_p}{\mu_p} - f_{r^*} - f_{ps}(1 - f_p) \right] \quad (17) \quad {}_{466}$$

$$\zeta = \mu_p \left[-2f_{r^*} - f_{ps}(1 - f_p) \right]$$
(19) 470
(20) 471

$$\eta = \mu_p^2 f_{r^*} \tag{21}$$

while μ_p is given by (12).

Proof: If a primary packet is directly delivered to D_p , 474 it experiences the queuing delay at Q_p only. This happens 475 with a probability $1 - \epsilon = f_p / \mu_p$. However, if the packet is 476 forwarded to D_p through the relay link, it experiences the total 477 queuing delay at both Q_p and Q_r . Thus, the average delay is 478

$$\tau = (1 - \epsilon)\tau_p + \epsilon(\tau_p + \tau_r) = \tau_p + \epsilon\tau_r \tag{22}$$

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where τ_p and τ_r denote the average delays at Q_p and Q_r , 480 respectively. The arrival rates at Q_p and Q_r are given by λ_p 481 and $\epsilon \lambda_p$, respectively. Thus, applying Little's law [22] renders 482

$$\tau_p = N_p / \lambda_p, \quad \tau_r = N_r / \epsilon \lambda_p. \tag{23}$$

Substituting (23) in (22) renders τ exactly matching (14). 484

Proceeding with computing N_p , we make use of the fact 485 that Q_p is a discrete-time M/M/1 queue with arrival rate λ_p 486 and service rate μ_p . Thus, N_p is directly given by (15) through 487 applying the Pollaczek-Khinchine formula [23]. However, the 488 dependence of the arrival and service processes of Q_r on 489 the state of Q_p necessitates using a MGF approach [24] to 490 calculate N_r . The MGF of the joint lengths of Q_p and Q_r is 491 defined as 492

$$G(x, y) = \lim_{n \to \infty} \mathbb{E}\left[x^{\mathbf{Q}_p[n]} y^{\mathbf{Q}_p[n]}\right]$$
(24)

where \mathbb{E} denotes the statistical expectation operator. Following 494 the framework in [4] and [24], we get 495

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$$G(x, y) = (\lambda_p x + 1 - \lambda_p) \frac{B(x, y)G(0, 0) + C(x, y)G(0, y)}{yD(x, y)}$$
497 (25)

where 498

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$$\begin{array}{ll} & B(x,y) = x(y-1)f_{r^{*}} \\ & 500 & C(x,y) = xf_{r^{*}} - yf_{p} - y^{2}f_{ps}(1-f_{p}) + xy(\mu_{p} - f_{r^{*}}) \\ & 501 & D(x,y) = x - (\lambda_{p}x + 1 - \lambda_{p})[f_{p} + yf_{ps}(1-f_{p}) \\ & + x(1-\mu_{p})]. \end{array}$$

$$\begin{array}{l} (26) \end{array}$$

First, we compute the derivative of (25) with respect to y and 503 then, take the limit of the result when x and y tend to 1. This 504 verifies that N_r is given by (16). 505

Theorems 1 and 2 provide closed-form expressions for the 506 network performance metrics, throughput and delay. These 507 expressions are mainly functions of the outage probabilities 508 on various links in the network, namely, f_p , f_{ps} , f_{r^*} , and f_{s^*} . 509 In the following lines, we quantify these outage probabilities 510 for the different combinations of power allocation and node 511 selection policies. It is worth noting that f_p and f_{ps} are 512 related to the PU side. Therefore, they remain the same for all 513 combinations of power allocation and node selection policies 514 which are performed at the SUs side. Using (1), we have 515

516
$$f_p = \mathbb{P}\left[\mathbf{h}_p > \frac{2^{R_0} - 1}{P_0/N_0}\right] = e^{-\alpha/\sigma_p^2}$$
(27)

where $\alpha = \frac{2^{R_0}-1}{P_0/N_0}$. This follows from the Rayleigh fading assumption that renders $\mathbf{h}_p \sim \exp(\sigma_p^2)$. Similarly, 517 518

⁵¹⁹
$$f_{ps} = \mathbb{P}\left[\max_{i \in \{1, \dots, N\}} \mathbf{h}_{ps_i} > \alpha\right] = 1 - (1 - e^{-\alpha})^N.$$
 (28)

On the other hand, we shift our attention to the SU side to 520 calculate f_{r^*} and f_{s^*} . We analyze the four cases arising from 521 the proposed power allocation and relay selection policies in 522 the following order: (i) EP-BSL, (ii) EP-BPL, (iii) AP-BSL, 523 and (iv) AP-BPL. Towards this objective, we first note that 524 for each SU, its link qualities to D_p and D_s are statistically 525 independent. Furthermore, these links are independent of the 526

other (N-1) users' links. Thus, we are dealing with 2N 527 i.i.d. random variables, \mathbf{h}_{r_i} and \mathbf{h}_{s_i} , $\forall i \in \{1, \dots, N\}$. Each 528 of these variables is exponentially distributed with mean 1 as 529 a direct consequence of the Rayleigh fading model consid-530 ered. We begin with an analysis of the distributions of the 531 random variables involved in the derivations of f_{r^*} and f_{s^*} , 532 specifically, \mathbf{h}_{r^*} , \mathbf{h}_{I} , and \mathbf{h}_{s^*} . Finding these distributions is 533 fundamental to the mathematical derivations presented next. 534 Obviously, the distributions is dependent on the node selection 535 policy employed and hence, we present a separate analysis for 536 BSL and BPL in Appendices A and B, respectively. 537

For the ease of exposition, we define $a = \frac{2^{R_0} - 1}{P_{\text{max}}/N_0}$, $b = (2^{R_0} - 1)^{-1}$, and $\beta = 1 - e^{-a}$. The exponential integral 538 539 function, $E_1[.]$, is defined as $E_1[x] = \int_x^\infty (e^{-t}/t) dt$. 540 541

Lemma 1: For EP-BSL, f_{r^*} and f_{s^*} are given by

$$f_{r^*} = 1 - \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k \frac{e^{-ka}}{(1+k/b)}$$
(29) 542
$$f_{r^*} = 1 - \frac{R^N}{k}$$
(30) 542

$$f_{s^*} = 1 - \beta^{\prime *}$$
. (30) 543

Proof: See Appendix C. 544

Lemma 2: For EP-BPL, f_{r^*} is given by

$$f_{r^*} = \frac{N}{N-1} \sum_{k=1}^{N-1} \binom{N-1}{k-1} [I_1 - I_2].$$
(31) 546

where

$$I_1 = \sum_{m=0}^{k-1} {\binom{k-1}{m}} \frac{(-1)^m}{(N-k+m+1)}$$
(32) 54

$$I_{2} = \sum_{m=0}^{k-1} \sum_{\ell=0}^{N} {\binom{k-1}{m} \binom{N}{\ell} \frac{(-1)^{m+\ell} e^{-a\ell}}{(N-k+m+\ell/b+1)}}$$
(33) 546

On the other hand, f_{s^*} is given by

$$f_{s^*} = \gamma \left(1 - \beta^{N-1} \right) + (1 - \gamma) \left(1 - \beta^N \right)$$
 (34) 551

where

$$\gamma = \frac{\lambda_p (1 - f_p) f_{ps}}{(\mu_p - \lambda_p) f_{r^*}}.$$
(35) 55

Proof: See Appendix D. Lemma 3: For AP-BSL, f_{r^*} is given by

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$$f_{r^*} = \beta^N (1 - \beta^N) + N \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k$$
556

$$\times e^{-a(k+1)} \left[I_3 - I_4 \right]$$
 (36) 557

where

$$I_{3} = \frac{N-1}{k+1} \sum_{\ell=0}^{N-2} {\binom{N-2}{\ell} \frac{(-1)^{\ell}}{(\ell+1)}} e^{-a(\ell+1)}$$
(37) 55

$$I_{4} = \frac{a}{b}e^{ab}(N-1)\sum_{\ell=0}^{N-2} {\binom{N-2}{\ell}}(-1)^{\ell}e^{\frac{a(1+b+\ell)(k+1-b)}{b}}$$

$$[a(1+b+\ell)(k+1)]$$
560

$$\times E_1 \left\lfloor \frac{a(1+b+\ell)(k+1)}{b} \right\rfloor. \tag{38}$$

On the other hand, f_{s^*} is given by (30). 562 Proof: See Appendix E. 563



Fig. 2. The probability of transmission success on the relay link versus P_{max}/N_0 for AP-based schemes. (a) AP-BSL. (b) AP-BPL.

(43)

Lemma 4: For AP-BPL, f_{r^*} is given by 564

$$f_{r^*} = \sum_{k=1}^{N-1} \sum_{\ell=0}^{k-1} \sum_{m=0}^{N-2} \binom{N-1}{k-1} \binom{k-1}{\ell} \binom{N-2}{m}$$

 $\times \frac{(-1)^{N-1}N^{-1}[15-16]}{(N-k+\ell+1)} + \beta^{N-1}(1-\beta^{N})$ (39)

where

568 I₅ =
$$\frac{e^{-a(m+1)}}{(m+1)} \sum_{n=0}^{N-1} {\binom{N-1}{n}} \frac{(-1)^n e^{-a(n+1)}}{(n+1)}$$
 (40)
569 I₆ = $\sum_{n=0}^{N-1} {\binom{N-1}{n}} \frac{a(-1)^n e^{-a(m+n-t+2)}}{(t-n-1)} e^{tc} E_1 [t(a+c)]$
570 (41)

570

57

576

and the terms t and c are 571

572
$$t = b(N - k + \ell + 1) + n + 1$$
(42)

$$c = a \left[\frac{m+1}{b(N-k+\ell+1)} - 1 \right].$$

On the other hand, f_{s^*} is given by (34). 574 Proof: See Appendix F. 575

VI. NUMERICAL RESULTS

In this section, we validate the closed-form expressions 577 derived in the paper via comparing theoretical and numerical 578 simulation results. We investigate the system performance in 579 terms of the primary and secondary throughput as well as the 580 average primary packets' delay. In addition, we quantify the 581 average power consumption at the SUs. Furthermore, we con-582 duct performance comparisons between the four strategies 583 resulting from the proposed power allocation and SU selection 584 policies. Accordingly, we draw insights about the benefit of 585 employing the proposed power allocation schemes. We set 586 $P_0/N_0 = 10$ dB. Results are averaged over 10^6 time slots. 587

Theorems 1 and 2 provide closed-form expressions for 588 primary and secondary throughput as well as average queueing 589 delay for primary packets. Generic expressions have been 590 provided that work for any combination of power allocation 591 and node selection policies. These expressions are functions 592 of the probabilities of successful transmissions on relay and 593

secondary links, i.e., f_{r^*} and f_{s^*} . This fact has been thoroughly 594 addressed in the appendices, where the four different power 595 allocation and node selection policies have been analyzed. 596 We start by validating our theoretical findings through sim-597 ulations. Towards this objective, the analytical expressions 598 for f_{r^*} , derived in Appendix E and F, are compared to 599 their corresponding simulation results for both AP-BSL and 600 AP-BPL in Fig. 2. We set a target rate $R_0 = 1.5$ (bits/channel 601 use) and we choose $\sigma_p^2 = 0.25$. Fig. 2(a) shows a perfect 602 match of theoretical and simulation results for AP-BSL for 603 any number of SUs, N. However, for AP-BPL, Fig. 2(b) 604 shows a slight deviation between both results. This difference 605 is attributed to the relaxation of the constraint that $\mathbf{h}_{\mathrm{I}} < \mathbf{h}_{r^*}$ 606 in the derivation presented in Appendix F, where we treat $\mathbf{h}_{\rm I}$ 607 and \mathbf{h}_{r^*} as independent random variables. This constraint is an 608 immediate consequence of the node selection policy presented 609 in Section IV-B.2. The relaxation has been done for the sake 610 of mathematical tractability. Nevertheless, Fig. 2(b) shows that 611 the constraint relaxation has a minor effect on the obtained 612 closed-form expression for f_{r^*} . This validates our theoretical 613 findings. Fig. 2 show that f_{r^*} consistently increases as the 614 number of SUs increases for both AP-based schemes. This 615 behavior is also true for EP-based schemes and is attributed 616 to multi-user diversity gains obtained through increasing N. 617

We investigated the effect of varying N in Fig. 2. Without 618 loss of generality, the rest of the results are presented for 619 N = 2, $R_0 = 2$ (bits/channel use), and $\sigma_p^2 = 0.25$. 620 We proceed with presenting the throughput of the PU and 621 the SUs for all combinations of power allocation and node 622 selection policies in Fig. 3. In Fig. 3(a), we plot the maximum 623 achievable PU throughput, i.e., maximum achievable λ_p given 624 by (9) in Theorem 1, versus P_{max}/N_0 . AP-BPL is shown to 625 outperform all other schemes. In particular, AP-BPL increases 626 the PU's throughput by up to 30% compared to AP-BSL 627 and EP-BPL, and more than 100% compared to EP-BSL. 628 Moreover, it is evident that AP-based schemes outperform 629 EP-based schemes [7], irrespective of the node selection policy 630 employed. In Fig. 3(b), we plot the SU throughput versus 631 λ_p at $P_{\rm max}/N_0 = 7$ dB. For the same node selection policy, 632 the throughput region of the AP-based schemes is shown to 633 strictly contain that of the EP based scheme. Furthermore, 634 at every feasible λ_p for EP-BPL, higher SU throughput 635



Fig. 3. The throughput of the PU and SUs for all combinations of power allocation and node selection policies. (a) Maximum achievable PU throughput versus P_{max}/N_0 . (b) SU throughput versus λ_p .



Fig. 4. The average queueing delay of PU's packets for different combinations of power allocation and node selection policies. (a) Average primary packets' delay versus P_{max}/N_0 . (b) Average primary packets' delay versus λ_p .

is attained by AP-BPL. Thus, power adaptation expands
 the stable throughput region. This shows the superiority of
 AP-based schemes in both PU and SU throughput over their
 EP-based counterparts.

In Fig. 4, we study the average delay encountered by the PU 640 packets. We refrain from plotting the results corresponding to 641 EP-BSL to get a clear view of the comparison. EP-BSL yields 642 much worse delay than the other three strategies. We plot the 643 average primary packet delay versus P_{max}/N_0 in Fig. 4(a) 644 at $\lambda_p = 0.1$. As the available power resources increase, 645 i.e., $P_{\rm max}/N_0$ increases, delay decreases. We attain lower 646 average delay through power adaptation. As expected, AP-BPL 647 holds its position as the best scheme with respect to PU. 648 Furthermore, we investigate the fundamental throughput-delay 649 tradeoff in Fig. 4(b). We plot the average packet delay for the 650 PU versus its throughput at $P_{\text{max}}/N_0 = 5$ dB. Intuitively, when 651 a node needs to maintain a higher throughput, it loses in terms 652 of the average delay encountered by its packets. Given that the 653 system is stable, the node's throughput equals its packet arrival 654 rate. Thus, increased throughput means injecting more packets 655 into the system resulting in a higher delay. Furthermore, 656 Fig. 4(b) shows that strictly lower average PU delay is attained 657 via AP-based schemes compared to EP allocation in [7]. It can 658 also be noticed that AP-BPL is still in the leading position 659 among all schemes in terms of both throughput and delay. 660 Fig. 4 shows that at $P_{\text{max}}/N_0 = 5$ dB and $\lambda_p = 0.1$, AP-BPL 661

reduces the PU's average delay by up to 27% compared to AP-BSL, and 40% compared to EP-BPL. Moreover, we validate the obtained closed-form expressions for average PU delay via simulations. Theoretical and simulation results for AP-BSL perfectly coincide. However, for AP-BPL, the slight deviation between theory and simulations is attributed to the relaxation of the constraint $\mathbf{h}_{\rm I} < \mathbf{h}_{r^*}$.

Finally, we plot the average powers transmitted by the 669 SUs in Fig. 5, i.e., average P_{s^*} and P_{r^*} , normalized to N_0 , 670 versus P_{max}/N_0 . Clearly, the AP-based schemes consume 671 significantly less power than the EP assignment represented 672 by the 45° line. Power adaptation results approximately in 673 50% reduction in energy consumption at the SUs, compared to 674 equal power allocation, at $P_{\text{max}}/N_0 = 15$ dB. For the average 675 power transmitted on the link $s^* \rightarrow D_s$, the first intuition that 676 comes to mind is that AP-BSL policy results in the minimum 677 average power. However, this is only true at high $P_{\rm max}/N_0$ 678 values. It is noticed that the results corresponding to AP-BPL 679 show slightly less power consumption than that of AP-BSL 680 at low P_{max}/N_0 values. This behavior approximately holds 681 till $P_{\rm max}/N_0 = 10$ dB. This is attributed to the nature of 682 the proposed AP policy which sets s^* silent if its maximum 683 power constraint is not sufficient to satisfy the condition 684 of success (5). Since in AP-BSL, s^* always sees the best 685 link to D_s , the number of slots in which it remains idle is 686 less than that in AP-BPL. This yields a higher throughput 687



Fig. 5. Average SUs' transmitted power normalized to N_0 versus P_{max}/N_0 .

at the expense of slightly higher average transmitted power. The same argument holds for comparing selection policies on the link $r^* \rightarrow D_p$.

691 A. Discussion on the Assumptions

The above system analysis is performed under the assump-692 tion of fully-backlogged SUs. The motivation behind this 693 assumption is two-fold. First, backlogged SUs represent the 694 worst case scenario from the PU's point of view. Since we 695 consider cooperative communications, a portion of the PU's 696 data is delivered to its intended destination via the relay 697 link, i.e., $r^* \to D_p$. However, the transmission of secondary 698 packets causes interference to the relay link as indicated 699 earlier. This interference is persistent in case of backlogged 700 SUs. Therefore, our results can be considered as a lower bound 701 on the achievable performance of the PU, i.e., a lower bound 702 on throughput and upper bound on delay. Furthermore, the 703 backlogged SUs assumption mitigates the interaction between 704 the queues of the SUs. This renders the system mathematically 705 tractable. Nevertheless, stochastic arrivals to the SUs' queues 706 can still be considered and queues interaction can be tackled 707 using the dominant system approach originally introduced 708 in [26]. However, this is out of the scope of the paper. 709

It is worth noting that in the derivations corresponding 710 to BPL-based schemes, i.e., in Sections VII and VII of 711 the Appendix, we consider \mathbf{h}_{I} and \mathbf{h}_{r^*} independent random 712 variables. However, they are coupled through the constraint 713 $\mathbf{h}_{\rm I} < \mathbf{h}_{r^*}$. This constraint is an immediate consequence of 714 the BPL node selection policy. We relax this constraint to 715 render the problem mathematically tractable. Nevertheless, 716 we quantify the effect of relaxing this constraint on the 717 obtained closed-form expressions for f_{r^*} through numerical 718 simulation results presented in Section VI. 719

Finally, we assume that SUs perfectly sense the PU's activity. This assumption has been made to avoid adding further complexity to the analysis which might distort the main message behind the paper. Nevertheless, imperfect sensing has been studied extensively in the literature. Reference [27] presents a comprehensive survey of spectrum sensing techniques in cognitive radio networks.

VII. CONCLUSION

We discuss a power allocation policy for cognitive radio 728 networks with multiple relays and propose different relaying 729 protocols depending on the network utility function. The 730 effect of SU power adaptation on throughput and average 731 delay is thoroughly investigated. We derive the closed-form 732 expressions for the achieved throughput and average delay and 733 validate the results through numerical simulations. Dynami-734 cally adapting the transmission powers at the SUs according 735 to the channel conditions results in substantial improve-736 ment in primary and secondary throughput. The SUs under 737 EP-based schemes always transmit at maximum power. This 738 results in excessive interference on the relay link which is 739 not the case for the AP-based schemes. Power adaptation is 740 performed at the SUs to transmit with the minimum power 741 required for the successful transmission. To further benefit 742 the system, the SUs back-off if their maximum permissible 743 power is not sufficient to yield a successful transmission and 744 avoid guaranteed outage events. The back-off benefits the other 745 transmitting SU by reducing the incurred interference and 746 thereby, causes throughput increase. The AP-based schemes 747 are shown to reduce the average queuing delay encountered 748 by the PU packets compared to their EP-based counterparts. 749 We perform mathematical analysis of the proposed schemes 750 and show numerically that the AP-based schemes save energy; 751 and achieve higher throughput and lower delay simultaneously. 752

DISTRIBUTIONS OF
$$\mathbf{h}_{r^*}$$
, \mathbf{h}_{I} , and \mathbf{h}_{s^*} for BSL 754

Referring to the policy described in Section IV-B.1,

$$\mathbf{h}_{s^*} = \max_{i \in \{1, \dots, N\}} \mathbf{h}_{s_i}.$$
 (44) 75

Therefore, the probability density function (PDF) of \mathbf{h}_{s^*} is

$$\mathcal{P}_{\mathbf{h}_{s^*}}(h) = N e^{-h} (1 - e^{-h})^{N-1}, \quad h \ge 0.$$
 (45) 758

As indicated earlier, the fact that s^* has the best link to D_s 759 gives absolutely no information about its link quality to D_p 760 and hence, 761

$$\mathcal{P}_{\mathbf{h}_{\mathrm{I}}}(h) = e^{-h}, \quad h \ge 0.$$
 (46) 762

On the other hand,

$$\mathcal{P}_{\mathbf{h}_{r^{*}}}(h) = (N-1)e^{-h}(1-e^{-h})^{N-2}, \quad h \ge 0.$$
 (47) 764

We present a rigorous argument to prove that (47) 765 is true. Consider the 2N random variables represent-766 ing the link qualities of the N SUs to D_p and D_s . 767 The SU with the best link to D_s is selected to transmit a 768 packet of its own. This leaves (N-1) possible candidates for 769 relaying a primary packet to D_p . Among the (N-1) random 770 variables representing the link qualities of these candidates to 771 D_p , their maximum is selected. This maximum has one of the 772 following two possibilites. 773

• It is the second maximum of $\{\mathbf{h}_{r_i}\}_{i=1}^N$. This occurs only when the same SU has the best link to both D_p and D_s simultaneously. A specific SU has the best link to both destinations simultaneously with probability $1/N^2$. 777

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• It is the maximum of $\{\mathbf{h}_{r_i}\}_{i=1}^N$. This occurs whenever s^* is not the SU having the best link to D_p , which has a probability 1 - (1/N).

The average distribution corresponding to the two possibilities presented above with their respective probabilities is exactly the same as the distribution of a maximum of (N - 1) i.i.d. exponential random variables with means 1 each. This is an easy-to-show fact using order statistics arguments, omitted for brevity. The proof of (47) is then concluded.

790 APPENDIX B 791 DISTRIBUTIONS OF \mathbf{h}_{r^*} , \mathbf{h}_{I} , and \mathbf{h}_{s^*} for BPL

According to the policy described in Section IV-B2,

$$\mathbf{h}_{r^*} = \max_{i \in \{1, \dots, N\}} \mathbf{h}_{r_i}.$$
 (48)

(49)

(50)

Therefore, the PDF of \mathbf{h}_{r^*} is

95
$$\mathscr{P}_{\mathbf{h}_{r^*}}(h) = Ne^{-h}(1-e^{-h})^{N-1}, \quad h \ge 0.$$

796 On the other hand,

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$$\mathscr{P}_{\mathbf{h}_{s}^{*}}(h) = (N-1)e^{-h}(1-e^{-h})^{N-2}, \quad h \ge 0.$$

⁷⁹⁸ An argument similar to that used to derive the distribution of ⁷⁹⁹ \mathbf{h}_{r^*} in Appendix A is used to derive (50).

The SU with the best link to D_p is selected to relay a 800 primary packet. This eliminates the possibility that s^* has the 801 best link to D_p , i.e., $\mathbf{h}_{\mathbf{I}}$ can not be the maximum of $\{\mathbf{h}_{r_i}\}_{i=1}^N$ 802 In other words, \mathbf{h}_{I} can possibly be the *k*th order statistic of the *N* random variables $\{\mathbf{h}_{r_{i}}\}_{i=1}^{N}$, where k = 1, ..., N - 1. The 803 804 kth order statistic is by convention the kth smallest random 805 variable. It remains to note that after the selection of r^* , 806 the remaining (N-1) SUs possess equal probabilities of 807 having the best link to D_s . Consequently, $\mathbf{h}_{\rm I}$ is equally likely 808 to be any *k*th order statistic of $\{\mathbf{h}_{r_i}\}_{i=1}^N$, k = 1, ..., N - 1. Then, the average distribution of these order statistics is 809 810 given by 811

12
$$\mathcal{P}_{\mathbf{h}_{\mathrm{I}}}(h) = \frac{N}{N-1} \sum_{k=1}^{N-1} \binom{N-1}{k-1} e^{-h(N-k+1)} \times (1-e^{-h})^{k-1}, \quad h \ge 0.$$
(51)

814 APPENDIX C 815 DERIVATION OF f_{r^*} AND f_{s^*} FOR EP-BSL

Using (1) and (4) along with the description of power allocation and node selection policies provided in Sections IV-A1 and IV-B1, respectively, we have

$$f_{r^*} = \mathbb{P}\left[\mathbf{h}_{r^*} > a + \frac{\mathbf{h}_{\mathrm{I}}}{b}\right].$$
 (52)

820 Then, total probability theory implies that

$$f_{r^*} = \int_0^\infty \mathbb{P}\left[\mathbf{h}_{r^*} > a + \frac{h}{b}\right] \mathcal{P}_{\mathbf{h}_{\mathbf{I}}}(h) dh$$
(53)

Thus, (53) is readily solved via substituting by the distributions of the random variables \mathbf{h}_{I} and \mathbf{h}_{r^*} provided in (46) and (47), respectively. We first note that

$$\mathbb{P}\left[\mathbf{h}_{r^*} > w\right] = 1 - (1 - e^{-w})^{N-1}, \quad w \ge 0$$
 (54) 825

and then use (54) with $w = a + \frac{h}{b}$ in (53) to get

$$f_{r^*} = \int_0^\infty \left[1 - \left[1 - e^{-\left(a + \frac{h}{b}\right)} \right]^{N-1} \right] .e^{-h} dh.$$
 (55) 827

To solve this integration, we use the binomial theorem

$$\left[1 - e^{-\left(a + \frac{h}{b}\right)}\right]^{N-1} = \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k e^{-k\left(a + \frac{h}{b}\right)}.$$
 (56) 825

We substitute by (56) in (55). Then, the integral solution renders f_{r^*} as in (29).

At the SUs side, we depend on (1) and (3) to write

$$f_{s^*} = \mathbb{P}\left[\mathbf{h}_{s^*} > a\right] = 1 - \beta^N \tag{57}$$
⁸³³

which follows directly from (45). This verifies f_{s^*} in (30).

APPENDIX D 835

DERIVATION OF f_{r^*} and f_{s^*} for EP-BPL 836

We use the description of power allocation and node selection policies presented in Sections IV-A1 and IV-B2, respectively. Using (1) and (4), f_{r^*} is given by (52) which is the same as (53) through total probability theory. The distributions of \mathbf{h}_{r^*} and \mathbf{h}_{I} given by (49) and (51), respectively, are used to solve the integral in (53) using similar steps to that presented in Appendix C. This renders f_{r^*} as given in (31).

An SU transmits on the best link to D_s only when Q_r is 844 empty. Therefore, 845

$$f_{s^*} = \mathbb{P}\left[\left.\bar{\mathcal{O}}_{s^*}\right|\mathsf{B}\right]\mathbb{P}\left[\mathsf{B}\right] + \mathbb{P}\left[\left.\bar{\mathcal{O}}_{s^*}\right|\bar{\mathsf{B}}\right]\mathbb{P}\left[\bar{\mathsf{B}}\right]$$
(58) 846

where O_{S^*} denotes the outage event on the secondary link, and B denotes the event that Q_r is non-empty. A bar over an event's symbol denotes its complement. Little's theorem [22] implies that

$$\mathbb{P}\left[\mathsf{B}\right] = \gamma \tag{59} \quad \text{851}$$

where γ is given by (35). In (59), we use the arrival and service rates of Q_r presented on both sides of (13), respectively. Next, we compute the probability of packet success on the secondary link when Q_r is busy. From (1) and (3), we have

$$\mathbb{P}\left[\left.\bar{O}_{s^*}\right|\mathsf{B}\right] = \mathbb{P}\left[\left.\mathbf{h}_{s^*} > a\right|\mathsf{B}\right] = 1 - \beta^{N-1}. \tag{60}$$

This follows from the distribution of \mathbf{h}_{s^*} given by (50). On the other hand, if Q_r is empty, s^* transmits on the best link among $\mathbb{S} \to D_s$, i.e., $\mathbf{h}_{s^*} = \max_{i \in \{1,...,N\}} h_{s_i}$. Thus, we have 859

$$\mathbb{P}\left[\left.\bar{O}_{s^*}\right|\bar{\mathsf{B}}\right] = \mathbb{P}\left[\left.\mathbf{h}_{s^*} > a\right|\bar{\mathsf{B}}\right] = 1 - \beta^N. \tag{61}$$

We substitute by the results of (59), (60), and (61) in (58). ⁸⁶¹ This verifies that f_{s^*} is given by (34). ⁸⁶²

APPENDIX E 863 DERIVATION OF f_{r^*} AND f_{s^*} FOR AP-BSL 864

Using total probability theory, we write 865

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$$f_{r^*} = \mathbb{P}\left[\left.\bar{O}_{r^*}\right|O_{S^*}\right]\mathbb{P}\left[O_{S^*}\right] + \mathbb{P}\left[\left.\bar{O}_{r^*}\right|\bar{O}_{S^*}\right]\mathbb{P}\left[\bar{O}_{S^*}\right]$$
(62)

where O_{r^*} denotes the outage event on the relay link. In (62), 867 we take into account the fact that s^* remains silent if P_{max} 868 is not sufficient to satisfy (5). Therefore, we compute the 869 probability of a successful transmission on the relay link in 870 both cases of s^* activity, i.e., either active or silent. Thus, 871 from (5), we have 872

$$\mathbb{P}[\mathcal{O}_{s^*}] = \mathbb{P}[\mathbf{h}_{s^*} < a] = \beta^N.$$
(63)

This can directly be verified using the distribution of \mathbf{h}_{s^*} 874 presented in (45). In the event of a sure outage on the 875 secondary link, s^* refrains from transmission. We then plug 876 $P_{s^*} = 0$ into (6) and write 877

$$\mathbb{P}\left[\left.\bar{\mathcal{O}}_{r^*}\right| \left.\mathcal{O}_{s^*}\right] = \mathbf{P}[\mathbf{h}_{r^*} > a] = 1 - \beta^N.$$
(64)

This result is explained as follows. When s^* is silent, r^* is 879 selected to be the SU with the best link to D_p to enhance the 880 PU throughput. Thus, in this specific case, \mathbf{h}_{r^*} is the maximum 881 of N exponential random variables with means 1 each. This 882 renders $\mathbb{P}[\mathbf{h}_{r^*} > a] = 1 - \beta^N$. 883

On the other hand, when s^* is active, i.e., $\mathbf{h}_{s^*} \geq a_s$ 884 we choose P_{s^*} to be the value that meets (5) with equality and 885 plug it into (6). After some algebraic manipulation, we write 886

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$$\mathbb{P}\left[\left.\bar{\mathcal{O}}_{r^*}\right|\left.\bar{\mathcal{O}}_{s^*}\right] = \mathbb{P}\left[\mathbf{h}_{\mathrm{I}} \leq b\left(\frac{\mathbf{h}_{r^*}}{a} - 1\right)\mathbf{h}_{s^*}\right|\mathbf{h}_{s^*} \geq a\right].$$
 (65)

The first step towards solving (65) requires the computation 888 of $\mathbb{P}[\mathbf{h}_{I} \leq z\mathbf{h}_{s^*} | \mathbf{h}_{s^*} \geq a]$ for an arbitrary $z \geq 0$. Proceeding 889 with that, we have 890

$$\mathbb{P}[\mathbf{h}_{\mathrm{I}} \leq z\mathbf{h}_{s^*} | \mathbf{h}_{s^*} \geq a] = \frac{\mathbb{P}[\mathbf{h}_{\mathrm{I}} \leq z\mathbf{h}_{s^*}, \mathbf{h}_{s^*} \geq a]}{\mathbb{P}[\mathbf{h}_{s^*} \geq a]}.$$
 (66)

The numerator of (66) can be computed as follows. 892

$$\mathbb{P}[\mathbf{h}_{\mathrm{I}} \leq z\mathbf{h}_{s^*}, \mathbf{h}_{s^*} \geq a] = \int_a^\infty \int_0^{zy} \mathcal{P}_{\mathbf{h}_{\mathrm{I}}}(x) \mathcal{P}_{\mathbf{h}_{s^*}}(y) dx dy \quad (67)$$

The distributions of \mathbf{h}_{I} and \mathbf{h}_{s^*} are given by (46) and (45), 894 respectively, and we use the fact that \mathbf{h}_{I} and $\mathbf{h}_{s^{*}}$ are inde-895 pendent. This information, along with the binomial theorem, 896 is used to solve the double integral in (67). Thus, 897

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$$\mathbb{P}[\mathbf{h}_{\mathrm{I}} \le z\mathbf{h}_{s^*}, \mathbf{h}_{s^*} \ge a] = N \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^k e^{-a(k+1)} \times \left[\frac{1}{k+1} - \frac{e^{-az}}{z+k+1}\right]. \quad (68)$$

Furthermore, we know from (63) that 900

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$$\mathbb{P}\left[\bar{O}_{s^*}\right] = \mathbb{P}[\mathbf{h}_{s^*} \ge a] = 1 - \beta^N.$$
(69)

Then, we substitute by (68) and (69) in (66). Next, we use 902 total probability theory to write (65) as 903

$$\mathbb{P}\left[\left.\bar{O}_{r^*}\right|\bar{O}_{s^*}\right]$$

$$= \int_a^{\infty} \mathbb{P}\left[\mathbf{h}_{\mathrm{I}} \le b\left(\frac{w}{a}-1\right)\mathbf{h}_{s^*}\right|\mathbf{h}_{s^*} \ge a\right] \mathcal{P}_{\mathbf{h}_{r^*}}(w)dw \quad (70)$$

where $\mathcal{P}_{\mathbf{h}_{*}}(.)$ is given by (47). We then substitute by the result 906 of (66), with $z = b(\frac{w}{a} - 1)$, in (70). The solution of the 907 integral yields 908

$$\mathbb{P}\left[\left.\bar{O}_{r^*}\right|\left.\bar{O}_{s^*}\right]\right.$$
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$$= \frac{N}{\left(1-\beta^{N}\right)} \sum_{k=0}^{N-1} \binom{N-1}{k} (-1)^{k} e^{-a(k+1)} \left[I_{3}-I_{4}\right]$$
(71) 910

where I_3 and I_4 are given by (37) and (38), respectively. The 911 derivation of (38) depends on the fact that 912

$$\int_{a}^{\infty} \frac{e^{-tw}}{w+c} dw = e^{tc} E_1[t(a+c)]$$
(72) 913

for any constants t and c. Substituting by (37) and (38) 914 in (71), and using (63), (64), (69), and (71) in (62), f_{r^*} is 915 shown to be given by (36). 916

For the SUs, f_{s^*} is shown to be given by (57) following the 917 same proof provided for the case of EP-BSL in Appendix C. 918

DERIVATION OF f_{r^*} AND f_{s^*} FOR AP-BPL

The derivation of f_{r^*} for AP-BPL follows the same foot-921 steps of the derivation presented in Appendix E. However, 922 the difference in the node selection policies induces different 923 distributions for the random variables of interest. We can write 924 f_{r^*} as in (62). First, we derive the first term in the RHS of (62) 925 as follows. 926

$$\mathbb{P}\left[\mathcal{O}_{s^*}\right] = \mathbb{P}[\mathbf{h}_{s^*} < a] = \beta^{N-1}. \tag{73}$$

This follows from the distribution of \mathbf{h}_{s^*} presented in (50). 928 When s^* is silent, we plug $P_{s^*} = 0$ into (6) and write 929

$$\mathbb{P}\left[\left.\bar{O}_{r^*}\right|O_{s^*}\right] = \mathbb{P}[\mathbf{h}_{r^*} > a] = 1 - \beta^N \tag{74}$$

where the distribution of \mathbf{h}_{r^*} is given by (49). Then, we shift 931 our attention to the second term in the RHS of (62). When 932 s^* is active, i.e., $\mathbf{h}_{s^*} \ge a$, we choose P_{s^*} to be the value that 933 meets (5) with equality and plug it into (6). Then, we compute 934 the probability of success on the relay link given that s^* is 935 active as in (65). We solve (67) using the distributions of \mathbf{h}_{s^*} 936 and \mathbf{h}_{I} in (50) and (51), respectively, along with the fact that 937 they are independent to get 938

$$\mathbb{P}\left[\mathbf{h}_{I} \leq z\mathbf{h}_{s^{*}}, \mathbf{h}_{s^{*}} \geq a\right] = \sum_{k=1}^{N-1} \sum_{\ell=0}^{k-1} \sum_{m=0}^{N-2} \binom{N-1}{k-1} \binom{k-1}{\ell} \binom{N-2}{m} \times \frac{N(-1)^{m+\ell}}{(N-k+\ell+1)} = 4 \times \left[\frac{e^{-a(m+1)}}{(N-k+\ell+1)} - \frac{e^{-a(m+z(N-k+\ell+1)+1)}}{(N-k+\ell+1)}\right]$$
(75) 94

(m+1) $(m+z(N-k+\ell+1)+1)$ for $z \ge 0$. Next, we substitute by the result of (75), with 942 $z = b(\frac{w}{a} - 1)$, in (70) and solve the integral. After some 943 algebraic manipulation, omitted for brevity, the second term 944 in the right hand side of (62) is found to be equal to

$$\sum_{k=1}^{N-1} \sum_{\ell=0}^{k-1} \sum_{m=0}^{N-2} \binom{N-1}{k-1} \binom{k-1}{\ell} \binom{N-2}{m}$$
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$$\times \frac{(-1)^{m+\ell} N^2}{(N-k+\ell+1)} [I_5 - I_6]$$
(76) 947

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¹⁹ I₅ =
$$\frac{e^{-a(m+1)}}{(m+1)} \int_{a}^{\infty} e^{-w} (1-e^{-w})^{N-1} dw$$
 (77)

$$I_{6} = \sum_{n=0}^{N-1} \frac{a(-1)^{n} e^{-a(m+n+2-t)}}{(t-n-1)} \int_{a}^{\infty} \frac{e^{-tw}}{w+c} dw$$
(78)

and the terms *t* and *c* are given by (42) and (43), respectively. The solution of the integral in (77) proves that I₅ is given by (40). We use (72) to show that I₆ is given by (41). Then, (73), (74), and (76) shows that f_{r^*} is given by (39).

On the other hand, f_{s^*} is shown to be given by (34) following the same proof provided for the case of EP-BPL in Appendix D.

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