

# Power-Efficient Spatial Multiplexing for Multiantenna MANETs

Diep N. Nguyen and Marwan Krunz  
Department of Electrical and Computer Engineering, University of Arizona  
E-mail: {dnnnguyen, krunz}@ece.arizona.edu

**Abstract**—We consider the problem of minimizing network’s transmit power for given transmission rate demands of all links in a multi-input multi-output (MIMO) ad hoc network. The problem is nonconvex, hence, challenging to be solved, even in a centralized manner. To derive a distributed solution, we reformulate the problem as a noncooperative game. We then propose a network interference function (NIF) that captures the total interference incurred at unintended receivers by all transmitters. The proposed NIF sets the light for designing transmitter-dependent pricing policies for the above game. A price-based iterative water-filling algorithm (PIWF) is proposed to find MIMO precoding matrices, which determines both beam directions and transmission power allocation among antennas (or data streams) at each transmitter. Simulations show that PIWF is more power-efficient than all existing MIMO precoding methods. Additionally, NIF under PIWF is also the least. Simulations also show the fast convergence of PIWF.

## I. INTRODUCTION

In this work, we focus on the power efficiency of multi-input multi-out (MIMO) mobile ad hoc network (MANETs). Specifically, we design transmitters’ precoding matrices so as to minimize the total network power under constraints on the transmission rates of all scheduled links. A precoding matrix determines both the power allocation among antennas and antennas’ radiation beam directions of a transmitter. Hence, our work not only addresses the power allocation but also optimizes beamformers for MIMO transmitters.

In the literature, there have been a vast body of works on MIMO precoding-matrix design, categorized into beamforming and generalized eigencoding. In beamforming e.g., [1] [2], there is only one data stream to be sent, hence all precoding matrices reduce to vectors (matrices of rank one). In generalized eigencoding, there is no constraint on the rank of the precoding matrices [3], i.e., several data streams can be sent simultaneously. Inspired by the introduction of the spatial multiplexing technique into existing networks (e.g., IEEE 802.11n allows up to four concurrent multiplexed streams), generalized eigencoding has recently attracted great attention. We emphasize that despite of the numerous MIMO precoding-matrix designs (e.g., [2] [4] [5] [6] and therein references), only few of them aim at minimizing network power consumption.

The authors of [1] and [3] respectively minimized the network power for the cases of beamforming and generalized eigencoding, where MIMO nodes are implicitly assumed to transmit and receive simultaneously on the same frequency band (full-duplex transceivers). The algorithms in [1] and [3] greatly outperform the greedy algorithm in [5] [7] in terms of power efficiency. These algorithms are also practically attractive as their inputs are two noise-plus-interference covariance matrices of the transmitter and the receiver, which can be obtained locally. However, in the case of half-duplex devices, we found that these algorithms do not always converge. Moreover, for convergent cases, they are surprisingly less power-efficient than the greedy algorithm. In [8], the power minimization problem was addressed for half-duplex MIMO nodes with beamforming technique. The channel in [8] is not reciprocal, hence transmit beamformers are selected from a finite set of codewords (codebook) to ease feedback requirement. The problem of power allocation and beamforming were treated in a sequential order. In contrast to previous works, we develop a

power-efficient generalized eigencoding method for half-duplex MIMO transceivers with reciprocal channels.

The joint optimization of power allocation for various data streams and beam patterns of MIMO nodes in MANETs is especially challenging, even in a centralized manner (due to the non-concavity of the objective function). To develop a distributed algorithm, we formulate the problem as a noncooperative game and propose a network interference function (NIF) that sets the light to design efficient transmitter-dependent pricing policies for the game.

Game theory has proved itself as a powerful tool for network resource allocation problems where each node/link acts as a player and maximizes its utility/payoff (e.g., transmission rate) by choosing its strategic response from its action space (e.g., transmit power, radiation pattern). In MIMO MANETs, each node independently selects its transmit strategy by adjusting the precoding matrix [5] [7] (we refer to this case as the greedy method). The per-user utility function is concave, resulting in a water-filling solution that converges to a Nash Equilibrium (NE) under some mild conditions. This NE is often not Pareto optimal [9], and may be inefficient with respect to the total network utility. This is due to the selfishness of the individual players, who attempt to maximize their own payoffs. It is well-known that by introducing a tax/pricing policy, the the social welfare is often improved [10].

In the context of network resource allocation, price often refers to the interference a transmitting node induces on others. We model the total network interference as the sum of traces of noise-plus-interference covariance matrices at all receivers. The price function for each transmitter is then set as the interference this transmitter brings to all unintended receivers. Using this price function, we propose a price-based iterative water-filling (PIWF) algorithm that minimizes not only the required transmit power but also the network interference. PIWF algorithm conserves more than 53% and 47% total transmission power, compared with the power-efficient precoding method in [3] (for convergent cases) and the greedy method in [5] [8], respectively. Our new definition of network interference allows the price function at a node is calculated locally by taking advantage of the symmetry in the channel gain. This supports the instant incorporation of PIWF to existing MIMO networks without incurring additional signaling overhead.

In Section II, we present the network model, problem formulation, and the PIWF algorithm. We analyze the properties of this algorithm in Section III. Numerical results are presented in Section IV. Finally, concluding remarks are provided in Section V.

## II. PROBLEM FORMULATION

Consider a MIMO MANET, consisting of  $K$  transmitter-receiver pairs. For a given transmitter  $u$ , let  $d(u)$  denote the corresponding receiver. We assume that each node has a half-duplex radio then can not transmit and receive at the same time.  $S$  is the set of all transmitters. Each node (transmitter or receiver) is equipped with  $M$  antennas. The channel gain matrix  $H_{d(u),u}$  for link  $(u, d(u))$  is under flat, Rayleigh fading, i.e., the elements of  $H_{d(u),u}$  are complex Gaussian random variables of zero mean and unit variance. Let  $(\cdot)^H$  denote the Hermitian

transpose,  $\text{tr}(\cdot)$  the trace of a matrix, and  $\det(\cdot)$  the determinant. The received signal is given by vector  $y_{d(u)}$  at  $d(u)$ :

$$y_{d(u)} = H_{d(u),u} \tilde{T}_u x_u + \sum_{i \in S \setminus \{u\}} H_{d(u),i} \tilde{T}_i x_i + N \quad (1)$$

where  $x_u$  is the vector of information symbols from node  $u$ ,  $N$  is the complex Gaussian noise vector with identity covariance matrix  $I$ , and  $\tilde{T}_u$  is the transmit precoding matrix at transmitter  $u$ .

The Shannon channel rate from transmitter  $u$  to its receiver  $d(u)$  is given by:

$$R_{d(u),u} = \log \det(I + \tilde{T}_u^H H_{d(u),u}^H C_{d(u)}^{-1} H_{d(u),u} \tilde{T}_u) \quad (2)$$

where  $C_{d(u)}$  is the noise-plus-interference covariance matrix at  $d(u)$ , given by:

$$C_{d(u)} = I + \sum_{i \in S \setminus \{u\}} H_{d(u),i} \tilde{T}_i \tilde{T}_i^H H_{d(u),i}^H.$$

As in [5], from the network's perspective, we want to minimize the total energy while maintaining a given transmission rate requirement for each link. Formally, the optimization problem can be stated as follows:

$$\begin{aligned} & \underset{\tilde{T}_u}{\text{minimize}} \quad \sum_{u \in S} \text{tr}(\tilde{T}_u \tilde{T}_u^H) \\ & \text{s.t.} \quad R_{d(u),u} \geq c_u, \quad \forall u \in S \end{aligned} \quad (3)$$

where  $c_u$  is the rate demand for link  $(u, d(u))$ .

In general, the above problem is not convex. In [3], instead of solving (3), the authors defined a new objective function, called the Total Interference Function (TIF). Minimizing TIF may lead to a local optimal solution of (3) (shown via simulations). However, TIF is not applicable to our setup where each user has only one half-duplex transceiver.

To address the problem in (3), we first formulate it as a noncooperative game. Using game theory notations, we define the utility function at transmitter  $u$  given its choice of the precoding matrix  $\tilde{T}_u$  as:  $U_u(\tilde{T}_u, \tilde{T}_{-u}) = -\text{tr}(\tilde{T}_u \tilde{T}_u^H)$ , where  $\tilde{T}_{-u}$  denotes the set of precoding matrices of all transmitters except  $u$ . Given the action space (defined below) and other transmitters' choices of their precoding matrices  $\tilde{T}_{-u}$ , the transmitter  $u$  maximizes its utility as follows:

$$\begin{aligned} & \underset{\tilde{T}_u}{\text{maximize}} \quad U_u(\tilde{T}_u, \tilde{T}_{-u}) \\ & \text{s.t.} \quad R_{d(u),u} \geq c_u. \end{aligned} \quad (4)$$

The constraints in (4) define the action space of the player/transmitter  $u$ . It is easy to verify the convexity of this per-user optimization problem (e.g., finding the Hessian of the objective and constraints by taking derivative with respect to the matrix  $\tilde{T}_u$ ). If all transmitters independently solve problem (4) (referred to as the greedy case), after some iterations, they all converge to a NE from which all players have no incentive to deviate. To push the NE closer to the Pareto curve, we introduce a pricing policy that makes players/transmitters more *socially responsible*. The utility function with price becomes:

$$U'_u(\tilde{T}_u, \tilde{T}_{-u}) \triangleq U_u(\tilde{T}_u, \tilde{T}_{-u}) - F_u(\tilde{T}_u)$$

where  $F_u(\tilde{T}_u)$  is the pricing function.

Deriving an optimal pricing policy (i.e., leading to the globally optimal point of the centralized problem (3)) for a game is often difficult. Hence, the pricing functions in the literature are often heuristic in nature. In order to derive a more efficient pricing policy, one may force the solution resulted from all distributed per-user optimization problems in (4) to converge to a local optimal solution of the network-wide problem (3). This can be realized by using the KKT conditions [11] to equate the stationary points of (4) to the stationary points of (3) (examples

of the application of this approach can be found in [12] for cognitive radio networks and [13] for MIMO MANETs). To ease the complexity of this procedure, the pricing functions are often in linear forms [14]. However, in our case, following this procedure leads to a pricing function that is dependent on global information and the Lagrangian multipliers ( $\mu_i$ ) of (3). Specifically, similar to [13], we can set the pricing function to  $\text{tr}(\tilde{T}_u A_u \tilde{T}_u^H)$ , where  $A_u$  is interpreted as the pricing-factor matrix at the transmitter  $u$ . After deriving the KKT conditions for (4) and (3),  $A_u$  is then computed as follows (the details of the mathematical manipulations are omitted):

$$\begin{aligned} A_u &= \mu_u H_{d(u),u}^H (C_{d(u)} + \tilde{T}_u^H H_{d(u),u}^H H_{d(u),u} \tilde{T}_u)^{-1} H_{d(u),u} \\ &+ \sum_{i \in S \setminus \{u\}} \mu_i H_{d(i),u}^H (C_{d(i)}^{-1} - (C_{d(i)} + \tilde{T}_i^H H_{d(i),i}^H H_{d(i),i} \tilde{T}_i)^{-1}) H_{d(i),u}. \end{aligned}$$

It can be seen that  $A_u$  is a function of not only the Lagrangian's multipliers  $\mu_i$  of (3) but also all precoding matrices of other transmitters. Hence, we have to find other ways to derive the pricing function.

The idea of characterizing and minimizing the total network interference of MIMO MANETs was first introduced in [1] for the case of beamforming and then extended to generalized eigencoding in [3]. Network interference models in [1] and [3] are generalized forms of the total squared correlation function in CDMA systems [15]. As mentioned earlier, these models, implicitly developed for full-duplex devices, lead to unstable and power-inefficient precoders for half-duplex MIMO transceivers.

We propose to quantify the network interference by the trace of all interference-plus-noise covariance matrices at all receivers (referred to as Network Interference Function-NIF):

$$\begin{aligned} \text{NIF} &\triangleq \text{tr}\left\{ \sum_{u \in \{S\}} C_{d(u)} \right\} \\ &= \text{tr}\left\{ KI + \sum_{d(i)} \sum_{u \in S \setminus \{i\}} H_{d(i),u} \tilde{T}_u \tilde{T}_u^H H_{d(i),u}^H \right\} \\ &= K \text{tr}(I) + \sum_{d(i)} \sum_{u \in S \setminus \{i\}} \text{tr}\left\{ \tilde{T}_u^H H_{d(i),u}^H H_{d(i),u} \tilde{T}_u \right\} \\ &= K \text{tr}(I) + \sum_{u \in S} \text{tr}\left\{ \tilde{T}_u^H \left[ \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u} \right] \tilde{T}_u \right\}. \end{aligned} \quad (5)$$

From (5), we observe that if each transmitter tries to minimize the interference from itself to all unintended receivers, the trace of matrix NIF is then minimized. Intuitively, the transmitter can realize that by selecting its appropriate precoding matrix such that the antenna's radiation beams are kept away as much as possible from unintended receivers. Hence, we propose a pricing function for transmitter  $u$  of the following form:

$$F_u(\tilde{T}_u) = \text{tr}\left\{ \tilde{T}_u^H \left[ \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u} \right] \tilde{T}_u \right\}. \quad (6)$$

The per-user optimization problem (4) at transmitter  $u$  becomes:

$$\begin{aligned} & \underset{\tilde{T}_u}{\text{minimize}} \quad \text{tr}\left\{ \tilde{T}_u^H \left[ I + \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u} \right] \tilde{T}_u \right\} \\ & \text{s.t.} \quad \log \det(I + \tilde{T}_u^H H_{d(u),u}^H C_{d(u)}^{-1} H_{d(u),u} \tilde{T}_u) \geq c_u. \end{aligned} \quad (7)$$

Note that to obtain its pricing function, a transmitter needs to know the channel matrix from itself to other receivers in its neighborhood. This information can be obtained easily by overhearing signalling packets at the MAC layer. For example, in the IEEE 802.11n scheme, by capturing the CTS message (containing a training sequence) from an unintended receiver, a transmitter can estimate the channel gains between itself and the receiver, assuming the channel-gain reciprocity (the channel matrix from  $u$  to  $d(i)$  is equal to the transpose of the channel

matrix from  $d(i)$  to  $u$ , i.e.,  $H_{d(i),u} = H_{d(i),u}^T$ ). The mechanism above does not incur additional cost to existing MIMO systems as training sequences are often used to estimate the channel between any pair of transmitter and receiver. Even in the case of fast fading (which results in difficulties to estimate the channel matrices), one may adapt the pricing function to take advantage of only the channel statistics. This is left for a future work.

The game in (7) can be solved iteratively by the following price-based iterative water-filling (PIWF) algorithm:

- 1) *Initialization*: Each transmitter  $u$  randomly selects its transmit precoding  $\tilde{T}_u^{(initial)}$  matrix and begins transmitting.
- 2) *Loop Until Convergence*: At the  $n$ th step, each transmitter  $u$  first updates its pricing function  $F_u(\tilde{T}_u)$  as in (6) by estimating the channel matrix  $H_{d(i),u}$  from itself to other receivers within its range. The noise-plus-interference covariance matrix  $C_{d(u)}$  at receiver  $d(u)$  is estimated locally and is fed back to its transmitter  $u$ . Subsequently, transmitter  $u$  updates its new precoding matrix  $\tilde{T}_u^{(n)}$  as the generalized eigen matrix of the matrix  $[I + \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u}]$  and matrix  $H_{d(u),u}^H C_{d(u)}^{-1} H_{d(u),u}$  (equation (8)). The new power allocation follows the water-filling algorithm derived in equation (10). Eventually,  $u$  transmits using its newly updated  $\tilde{T}_u^{(n)}$ .

The iteration in the loop can be of Jacobi or Gauss-Seidel types. Using Jacobi iteration type, at the  $n$ th step,  $C_{d(u)}$  is updated with  $\tilde{T}_{-u}$  from the previous iteration of all other transmitters. Under the Gauss-Seidel update method, the noise-plus-covariance matrix  $C_{d(u)}$  at the receiver  $d(u)$  is computed based on the latest  $\tilde{T}_{-u}$  matrices from other transmitters (thus, some precoding matrices are from the previous iteration and some from the current iteration).

### III. ANALYSIS OF THE PIWF ALGORITHM

In this section, we prove the existence of a NE for the PIWF algorithm. The convergence of this algorithm is established under the Gauss-Seidel update method. We begin by showing how to obtain the best response for each player by solving (7).

**Theorem 1:** The best response, i.e., the optimal precoding matrix  $\tilde{T}_u$  at node  $u$  for problem (7), is the generalized eigen matrix of the matrix  $[I + \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u}]$  and matrix

$$H_{d(u),u}^H C_{d(u)}^{-1} H_{d(u),u}.$$

*Proof:* The proof uses Hadamard's inequality as in [3] [13] and is omitted due to space limit. ■

Theorem 1 states a class of matrices that the solution  $\tilde{T}_u$  of (7) must belong to. This class tells the directions that a transmitter  $u$  should point its radiation beams to. The next step is to find the optimal power allocation. Let  $P_u$  denote the power allocation matrix of all streams sent from  $u$  to  $d(u)$  with  $P_u(k, k)$  be the power allocated on data stream  $k$  ( $k$  is from 1 to  $M$ ). To ensure that  $\tilde{T}_u$  belongs to the class of matrices specified by theorem 1, let:

$$\tilde{T}_u = T_u P_u^{1/2} \quad (8)$$

where  $P_u^{1/2}$  is the square root of matrix  $P_u$  and  $T_u$  is a unit-norm matrix, obtained by normalizing columns of the generalized eigen matrix of matrices  $[I + \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u}]$  and

$$H_{d(u),u}^H C_{d(u)}^{-1} H_{d(u),u}.$$

Since  $T_u$  is also a generalized eigen matrix of  $[I + \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u}]$  and matrix  $H_{d(u),u}^H C_{d(u)}^{-1} H_{d(u),u}$ ,  $T_u$  diagonalizes matrix  $[I + \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u}]$  and matrix

$$H_{d(u),u}^H C_{d(u)}^{-1} H_{d(u),u}:$$

$$\begin{aligned} \tilde{T}_u^H [I + \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u}] \tilde{T}_u &= P_u^{1/2} D_u^{(1)} P_u^{1/2} \\ \tilde{T}_u^H H_{d(u),u}^H C_{d(u)}^{-1} H_{d(u),u} \tilde{T}_u &= P_u^{1/2} D_u^{(2)} P_u^{1/2} \end{aligned}$$

where  $D_u^{(1)}$  and  $D_u^{(2)}$  are diagonal matrices with nonnegative entries  $D_u^{(1)}(k, k)$  and  $D_u^{(2)}(k, k)$  along their diagonals, respectively. Problem (7) becomes:

$$\begin{aligned} &\underset{P_u(k,k)}{\text{minimize}} \sum_{k=1}^M P_u(k, k) D_u^{(1)}(k, k) \\ &\text{s.t.} \quad \sum_{k=1}^M \log(1 + P_u(k, k) D_u^{(2)}(k, k)) \geq c_u. \end{aligned} \quad (9)$$

The solution to (9) is obtained by a using water-filling algorithm (interested readers are referred to [3] for more details), as follows:

$$P_u(k, k) = \max \left\{ 0, \left[ \frac{w_u}{D_u^{(1)}(k, k)} - \frac{1}{D_u^{(2)}(k, k)} \right] \right\} \quad (10)$$

where  $w_u$  is the water-level on  $m$  parallel data streams, given by:

$$w_u = \exp \left( \frac{c_u - \sum_{k=1}^m [\log(D_u^{(2)}(k, k)) - \log(D_u^{(1)}(k, k))]}{m} \right).$$

Note that, from (10), the effect of the pricing function in (6) is now implicitly embedded in the eigenvalues  $D_u^{(1)}(k, k)$  ( $k = 1 \dots M$ ). When the price on sub-channel  $k$  is high (large  $D_u^{(1)}(k, k)$ ), less power is allocated for it, and viceversa.

**Theorem 2:** There exists at least one NE for the non-cooperative game in (7).

*Proof:* We need to show that:

- 1) The action space of each player is convex and compact.
- 2) The utility function  $U_u(\tilde{T}_u, \tilde{T}_{-u})$  is concave.

To prove the first requirement, we impose a technical constraint on the total power consumption of the transmitter  $u$ , meaning that  $\text{tr}(\tilde{T}_u^H \tilde{T}_u) \leq P_{\max}$ . This assumption is reasonable as MANET nodes' power is always limited. This assumption also implicitly eliminates rate demands that are infeasible/unattainable as a results of interference-limited communications. As shown previously, given the matrices  $[I + \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u}]$  and  $H_{d(u),u}^H C_{d(u)}^{-1} H_{d(u),u}$ , the action space of the transmitter  $u$  reduces to finding the power allocation vector  $P_u$ . Now, the constraint on the total power of a node becomes  $\sum_k P_u(k, k) \leq P_{\max}$ . This is a linear constraint, hence is convex. The other constraint is  $[c_u - \log \det(I + \tilde{T}_u^H H_{d(u),u}^H C_{d(u)}^{-1} H_{d(u),u} \tilde{T}_u)] \leq 0$  is also a convex function of  $\tilde{T}_u$  while fixing  $\tilde{T}_{-u}$ . Therefore, the feasible region of (7) or the action space of the game is the intersection of convex regions, thus convex. Its compactness is due to the constraint on the total power consumption.

To check the concavity of the objective function  $U_u(\tilde{T}_u, \tilde{T}_{-u})$ , we prove that  $-U'_u(\tilde{T}_u, \tilde{T}_{-u})$  is convex by showing that the Hessian of  $-U'_u(\tilde{T}_u, \tilde{T}_{-u})$  is positive-definite, as follows:

$$\begin{aligned} \mathbf{H} &\triangleq \frac{\partial^2 U'_u(\tilde{T}_u, \tilde{T}_{-u})}{\partial \tilde{T}_u^2} \\ &= I + \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u} \succ 0. \end{aligned} \quad (11)$$

where  $B \succ 0$  means that matrix  $B$  is positive-definite. Hence, the game in (7) is a concave game which always admits a fixed point. ■

The existence of the NE does not mean that PIWF converges. Fortunately, if we use the Gauss-Seidel iteration method, we can claim the convergence of the PIWF algorithm to a NE.

**Theorem 3:** The PIWF algorithm that solves the problem (7) at each user converges to a NE.

*Proof:* Let's define a Lyapunov-type function of all precoding matrices as follows:

$$\begin{aligned} L &\triangleq \sum_{u \in S} -U'_u(\tilde{T}_u, \tilde{T}_{-u}) \\ &= \sum_{u \in S} \text{tr}\{\tilde{T}_u^H [I + \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u}] \tilde{T}_u\}. \end{aligned} \quad (12)$$

At the  $n$ th iteration step :

$$L^{(n)} = \sum_{u \in S} \text{tr}\{\tilde{T}_u^{(n)H} [I + \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u}] \tilde{T}_u^{(n)}\} \quad (13)$$

where  $\tilde{T}_u^{(n)}$  is the precoding matrix of the transmitter  $u$  at the  $n$ th iteration. We have the following observations:

**Observation 1:** For each transmitter  $u \in S$ ,  $L^{(n)}$  is a convex function of  $\tilde{T}_u^{(n)}$ .

This observation is due to the fact that the Hessian of  $L^{(n)}$  with respect to  $\tilde{T}_u^{(n)}$  is that of  $-U'_u(\tilde{T}_u^{(n)}, \tilde{T}_{-u}^{(n-1)})$ , previously shown to be positive-definite. Hence:

$$\begin{aligned} L^{(n-1)} &\geq L^{(n)} + \text{tr}\left\{\left[\frac{\partial L}{\partial \tilde{T}_u} \Big|_{\tilde{T}_u = \tilde{T}_u^{(n)}}\right]^H (\tilde{T}_u^{(n-1)} - \tilde{T}_u^{(n)})\right\} \\ &\geq L^{(n)} + \text{tr}\left\{\left[\frac{\partial L}{\partial \tilde{T}_u} \Big|_{\tilde{T}_u = \tilde{T}_u^{(n)}}\right]^H \tilde{T}_u^{(n-1)}\right\} \\ &\quad - \text{tr}\left\{\left[\frac{\partial L}{\partial \tilde{T}_u} \Big|_{\tilde{T}_u = \tilde{T}_u^{(n)}}\right]^H \tilde{T}_u^{(n)}\right\}. \end{aligned} \quad (14)$$

**Observation 2:**

$$\begin{aligned} &-U'_u(\tilde{T}_u^{(n)}, \tilde{T}_{-u}^{(n-1)}) \\ &= \text{tr}\{\tilde{T}_u^{(n)H} [I + \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u}] \tilde{T}_u^{(n)}\} \\ &= \text{tr}\left\{\left([I + \sum_{d(i) \neq d(u)} H_{d(i),u}^H H_{d(i),u}]\tilde{T}_u^{(n)}\right)^H \tilde{T}_u^{(n)}\right\} \\ &= \text{tr}\left\{\left[\frac{\partial L}{\partial \tilde{T}_u} \Big|_{\tilde{T}_u = \tilde{T}_u^{(n)}}\right]^H \tilde{T}_u^{(n)}\right\}. \end{aligned}$$

Moreover, using the Gauss-Seidel iteration method, we have:

$$\tilde{T}_u^{(n)} = \arg \max_{\tilde{T}_u} \left( U'_u(\tilde{T}_u, \tilde{T}_{-u}^{(n-1)}) \right) = \arg \min_{\tilde{T}_u} \left( -U'_u(\tilde{T}_u, \tilde{T}_{-u}^{(n-1)}) \right)$$

Hence:

$$\text{tr}\left\{\left[\frac{\partial L}{\partial \tilde{T}_u} \Big|_{\tilde{T}_u = \tilde{T}_u^{(n)}}\right]^H \tilde{T}_u^{(n)}\right\} \leq \text{tr}\left\{\left[\frac{\partial L}{\partial \tilde{T}_u} \Big|_{\tilde{T}_u = \tilde{T}_u^{(n-1)}}\right]^H \tilde{T}_u^{(n-1)}\right\}. \quad (15)$$

From (14) and (15), we get:

$$L^{(n-1)} \geq L^{(n)}.$$

In other words,  $L$  is a non-increasing function. Intuitively, at each iteration, transmitters solve the per-user optimization (7) to minimize their required transmit power and interference. Hence,  $L$ , which represents the total power and interference from all transmitters in the network at the next iteration, must be a non-increasing function.  $L$  is simply bounded from below by zero. Thus, PIWF algorithm must converge. The converged point must be a NE, otherwise one user can still unilaterally improve its return  $U'_i(\tilde{T}_i, \tilde{T}_{-i})$  (that violates the convexity of the individual problem (7)). ■

## IV. NUMERICAL RESULTS

In this section, the performance of the PIWF algorithm is evaluated via simulations using MATLAB. Specifically, we compare the transmission power and network interference of PIWF with two representative methods in the literature. The first method was developed similarly to that in [5] [7], where no pricing function was used and the precoding matrices were obtained from problem (4) by using an iterative water-filling algorithm (thus, we name this method as IWF). Note that the performance of IWF should be superior to that of [8] as IWF jointly solves the power allocation and beamforming problems while [8] addressed these problems sequentially. The second method is from [3], in which nodes were assumed to be full-duplex (hence, we refer to this method as FD) and the precoding matrices were found to satisfy the data rate requirement in the both forward and backward directions. From a game theory's point of view, the precoding matrices in [3] are obtained by introducing a pricing-function that depends on the covariance matrix of interference at the transmitter.

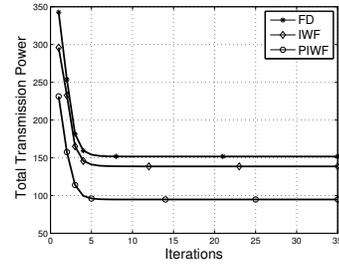


Fig. 2. Total transmission power under PIWF, IWF and FD algorithms v.s. iterations.

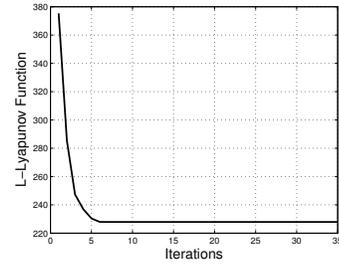


Fig. 3. PIWF's convergence: Lyapunov-type function  $L$  (equation (12)) v.s. iterations.

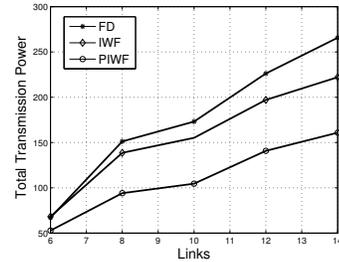


Fig. 4. Total transmission power under PIWF, IWF, and FD algorithms v.s. the number of links.

The simulated MIMO MANET consists of  $K$  pairs of transmitters and receivers, uniformly distributed within an area of 1000 meter x 1000 meter. Each node uses an antenna array of 4 elements to transmit or receive. The free-space attenuation factor is 4 and the received power at a reference distance of 100 meters reduces 10dB compared to the transmit power. The data rate requirement for every link is set to 1bps/Hz. The maximum spreading of arrival angle of signal at a receiver is  $\pi/6$ . To account for the randomness, for each plotted point, we generate 50 different network topologies and take the averaged

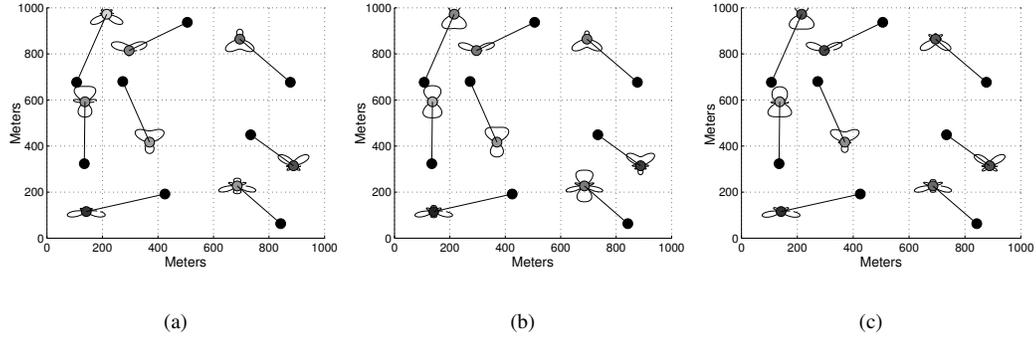


Fig. 1. Snapshots of an 8-link network and its antennas' radiation patterns under FD (a), IWF (b), PIWF (c) algorithms.

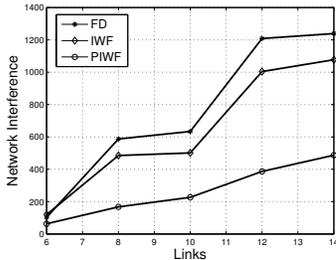


Fig. 5. Network interference under PIWF, IWF, and FD algorithms v.s. the number of links.

results. As algorithms in [3] does not always converge, its results are recorded for only convergent cases.

First, consider an 8-link network. A snapshot of its topology and antenna patterns under FD, IWF, PIWF algorithms are shown in Figure IV. As seen, transmitters under the PIWF algorithm tend to steer their beams away from nearby receivers. Figure 2 compares network's total power consumption of the three precoding methods. As we can see, PIWF conserves about 47% and 53% power, compared with IWF and FD, respectively. With half-duplex transceivers, it is worth noting that IWF is slightly more power efficient than FD (though under the full-duplexity assumption, it has been known that FD greatly outperforms IWF [3]). We also observe that PIWF converges after about 4 iterations, faster than FD and IWF (which converges after about 7 iterations). The fast convergence of PIWF facilitates its distributed protocol implementation.

The Lyapunov-type function  $L$ , defined in (12), is plotted in Figure 3. As can be seen,  $L$  is a non-increasing function.

Figure 4 depicts the total transmission power of the three algorithm versus the number of links in the network. As expected, PIWF requires less power than both IWF and FD. For all three precoding methods, when the number of links increases, transmitters have to spend more power to combat higher multi-user interference. Consequently, network interference NIF also increases with the number of network's links, as shown in Figure 5. Figure 5 also shows that NIF under PIWF algorithm is the least among three algorithms.

## V. CONCLUSIONS AND FUTURE WORK

In this work, we proposed a distributed algorithm to design the precoding matrices for MIMO MANETs with half-duplex transceivers using generalized eigencoding. The objective of the algorithm, called PIWF, is to minimize the total network power while satisfying the transmission rate requirements of all links. The key idea in the PIWF algorithm is the introduction of a network interference function, then transmitter-dependent pricing functions. We prove the existence of a NE and the convergence of the algorithm to the NE. Simulation results

show that the algorithm significantly conserves total power consumption in the network, compared with existing methods, e.g., those in [5] [8] [3] [7]. Our algorithm can be implemented distributively without incurring additional signalling overhead. Our future work will focus on extending the algorithm to the case when only partial channel information is available.

## REFERENCES

- [1] R. Iltis, S.-J. Kim, and D. Hoang, "Noncooperative iterative MMSE beamforming algorithms for ad hoc networks," *IEEE Transactions on Communications*, vol. 54, no. 4, pp. 748–759, 2006.
- [2] C. Shi, R. Berry, and M. Honig, "Local interference pricing for distributed beamforming in MIMO networks," in *Proceedings of IEEE Military Communications Conference MILCOM*, 2009, pp. 1–6.
- [3] D. Hoang and R. Iltis, "Noncooperative eigencoding for MIMO Ad hoc Networks," *IEEE Transactions on Signal Processing*, vol. 56, no. 2, pp. 865–869, 2008.
- [4] C. Shi, D. Schmidt, R. Berry, M. Honig, and W. Utschick, "Distributed interference pricing for the MIMO interference channel," in *Proceedings of IEEE International Conference on Communications*, 2009, pp. 1–5.
- [5] S. Ye and R. Blum, "Optimized signaling for MIMO interference systems with feedback," *IEEE Transactions on Signal Processing*, vol. 51, no. 11, pp. 2839–2848, Nov. 2003.
- [6] G. Scutari, D. Palomar, and S. Barbarossa, "Competitive design of multiuser MIMO systems based on game theory: A unified view," *IEEE Journal on Selected Areas in Communications*, vol. 26, no. 7, pp. 1089–1103, 2008.
- [7] G. Arslan, M. Demirkol, and S. Yuksel, "Power games in MIMO interference systems," in *Proceedings of the International Conference on Game Theory for Networks*, May 2009, pp. 52–59.
- [8] E. Zeydan, D. Kivanc, U. Tureli, and C. Comaniciu, "Joint iterative beamforming and power adaptation for mimo ad hoc networks," *EURASIP Journal on Wireless Communications and Networking*, 2011.
- [9] M. J. Osborne, *An Introduction to Game Theory*. Oxford University Press, 2004.
- [10] J. Hirshleifer, A. Glazer, and D. Hirshleifer, *Price Theory and Applications: Decisions, Markets, and Information*. Cambridge University Press, 2005.
- [11] S. Boyd and L. Vandenberghe, *Convex optimization*. Cambridge University Press, 2004.
- [12] F. Wang, M. Krusz, and S. Cui, "Price-based spectrum management in cognitive radio networks," *IEEE Journal of Selected Topics in Signal Processing*, vol. 2, no. 1, pp. 74–87, 2008.
- [13] S.-J. Kim and G. Giannakis, "Optimal resource allocation for MIMO ad hoc cognitive radio networks," in *Proceedings of 46th Annual Allerton Conference on Communication, Control, and Computing*, 2008, pp. 39–45.
- [14] C. Saraydar, N. Mandayam, and D. Goodman, "Efficient power control via pricing in wireless data networks," *IEEE Transactions on Communications*, vol. 50, no. 2, pp. 291–303, Feb. 2002.
- [15] S. Ulukus and R. Yates, "Iterative construction of optimum signature sequence sets in synchronous cdma systems," *IEEE Transactions on Information Theory*, vol. 47, no. 5, pp. 1989–1998, Jul 2001.