A Study of Tradeoff between Energy Efficiency and Control Complexity for CDMA Wireless Sensor Networks

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***Abstract*— For CDMA-based WSNs, we quantitatively investigate and compare the optimal energy efficiencies and control complexities for three different power/time control (PTC) schemes: PTC with independent transmission power and time (PTC-IPT), PTC with unified transmission time (PTC-UT), and PTC with unified spreading gain (PTC-USG). These schemes provide different degrees of control and require different amounts of overhead. Under each scheme, the minimization of system's energy consumption is formulated as a non-convex optimization problem. The optimal transmission power and time are derived analytically through a *variable-decoupling* approach. The analytical nature of our results makes it feasible to compare the performance in closed form. Numerical examples and simulations are provided to validate our analysis.**

I. Introduction

To improve energy efficiency in wireless sensor networks (WSNs), recent studies proposed various channel-adaptive transmission techniques that exploit the flexibility of parameters such as packet size [1], coding rate [2], modulation rate [2], number of channels in an FDMA system [3], and slot length in a TDMA system [4]. In a previous work [6], we considered the use of CDMA for channel access in a WSN. This scheme allows sensors to transmit data simultaneously to a remote sink using different spreading (signature) codes. The optimal transmission power and time (or equivalently, the rate) at each node are derived via a sequential algorithm, which minimizes the total energy consumption of the network. While providing significant energy savings compared with existing control strategies, this approach requires extensive computation overhead because of the large number of variables that need to be determined. Because a typical WSN can only provide limited computing capabilities, control strategies that provide gracefully degraded energy efficiency but demand significantly smaller computation overhead may be more desirable than the energy-optimal but computation-extensive one. The limitation on control overhead motivates us to investigate the tradeoff between the energy efficiency and the control complexity of the transmission control strategy.

The contributions of this work are threefold. Firstly, to gain insight into the efficiency-complexity tradeoff for CDMA WSNs, three different schemes of joint transmit power and time control (PTC) are considered in our optimization framework: PTC with independent transmission power and time (PTC-IPT) [6], PTC with unified transmission time (PTC-UT), and PTC with unified spreading gain (PTC-USG). These schemes differ in the dependence between the control variables (i.e., the transmit power and time) and represent a full range of practical implementations with various control complexities for the joint transmit power/time control. Secondly, using a variable-decoupling approach, for each control scheme, we derive either an efficient sequential algorithm or a closed-form approximate solution for the optimal transmission power and time that minimizes the total energy. Thirdly, based on our analytical results, the energy efficiencies and control complexities associated with each scheme are quantitatively compared. The results indicate that energy efficiency is not sensitive to the difference in transmission time between individual sensor nodes, thus there may be no need to separately control the transmission time of each node.

The rest of the paper is organized as follows. The system model is described in Section II. The three transmission schemes and their optimization formulations are presented in Section III. The analytical solutions are given in Section IV. Performance comparisons between schemes are conducted in Section V. Numerical examples are given in Section VI and conclusions are provided in Section VII.

II. Model Description

We consider a DS-CDMA-based WSN that consists of a set of densely distributed sensor nodes. The nodes transmit their data to a remote sink in a one-hop WSN [4] or to a local cluster head in a clustered WSN [5]. In either scenario, the destination is a dedicated node of much more powerful battery and computing functionality than ordinary sensors. Let o denote this destination node and let N be the number of active sensors at any given time. The information from the N sensors is transmitted simultaneously over a spread-spectrum bandwidth of W Hz. The single-sided power spectrum density of the additive white Gaussian noise (AWGN) is N_0 Watt/Hz.

For node i, $i = 1, \ldots, N$, there are B_i bits in the queue waiting to be transmitted to node o using transmit power P_{ti} and for transmission duration T_i. Different transmission rates are supported by using variable spreading gains. Let the channel gain between nodes i and o be h_i and assume that the channel is stationary for the duration T_i. The QoS requirement of sensor i is presented by the triple $(\gamma_i, T_i^{limit}, P_{\max})$, where γ_i is the bit-energy-to-interference-ratio threshold for correct reception of sensor i's signal, $T_i^{limit} \geq T_i$ is an upper limit on the transmission delay, and $P_{\max} \geq P_{ti}$ is the maximum transmit power (assumed the same for all nodes).

For node i, the energy consumption (E_i) consists of a

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transmission component and a circuit component, i.e.,

$$E_i = (P_{ti} + P_{ci})T_i \tag{1}$$

where P_{ci} is the power consumed by the circuit at sensor i. Following a similar model to that in [2], P_{ci} can be written as

$$P_{ci} = \alpha_i + (\frac{1}{\eta} - 1)P_{ti} \tag{2}$$

where α_i is a transmit-power-independent component that accounts for the power consumed by the digital-to-analog converter, the signal filters, and the modulator; and η is the efficiency factor of the power amplifier. Substituting (2) into (1), the energy consumption of node i is given by

$$E_i = \frac{1}{\eta}P_{ti}T_i + \alpha_i T_i = \frac{1}{\eta}(P_{ti} + \alpha_{ciri})T_i \tag{3}$$

where $\alpha_{ciri} = \eta\alpha_i$ is the equivalent circuit power consumption. For N active sensor nodes, the total energy consumption is

$$E_{total} = \sum_{i=1}^{N} E_i = \frac{1}{\eta}\sum_{i=1}^{N}(P_{ti} + \alpha_{ciri})T_i. \tag{4}$$

III. Transmission Control Schemes

The primary objective of our work is to find the optimal transmission power P_{ti}^o and transmission time T_i^o for each node i such that the total energy consumed in transmitting $\sum_{i=1}^{N} B_i$ bits is minimized while the QoS requirement of each transmission is satisfied. We consider three joint power/time control schemes that provide different degrees of freedom in controlling power and time.

A. PTC-IPT Scheme

The PTC-IPT scheme provides complete freedom in controlling the transmission power and time of every node. For a WSN of N nodes, there are $2N$ independent control variables, i.e., P_{ti} and T_i, $i = 1, \ldots, N$, which need to be optimized to minimize the total energy consumption. More specifically, the optimization problem is formulated as

$$\begin{cases} \text{minimize}_{\{\mathbf{P}_t, \mathbf{T}\}} \sum_{i=1}^{N}(P_{ti} + \alpha_{ciri})T_i \\ s.t. \\ \left(\frac{E_b}{I_0}\right)_i \geq \gamma_i, \quad i = 1, \ldots, N \\ 0 \leq T_i \leq T_i^{limit}, \quad i = 1, \ldots, N \\ 0 \leq P_{ti} \leq P_{\max}, \quad i = 1, \ldots, N \end{cases} \tag{5}$$

where $\mathbf{P}_t \overset{\text{def}}{=} (P_{t1}, \ldots, P_{tN})$ is the transmit power vector, $\mathbf{T} \overset{\text{def}}{=} (T_1, \ldots, T_N)$ is the transmission time vector, and $\left(\frac{E_b}{I_0}\right)_i$ is the received bit-energy-to-interference-density-ratio at node o for sensor i:

$$\left(\frac{E_b}{I_0}\right)_i = \frac{W}{B_i}\frac{h_i P_{ti} T_i}{\delta\sum_{j=1, j\neq i}^{N} h_j P_{tj} + N_0 W} \tag{6}$$

where $R_i = \frac{B_i}{T_i}$ is the transmission rate under the assumption of BPSK modulation, h_i is the channel gain, and δ is the *orthogonality factor*, representing multiple access interference (MAI) from the imperfectly orthogonal spreading codes and the asynchronous chips across simultaneous transmitting nodes. Typical values for δ are $\frac{2}{3}$ and 1 for a chip of rectangular and sinoide shapes, respectively.

B. PTC-UT Scheme

Under PTC-UT scheme, node o specifies the N transmission powers and a common transmission time for all nodes. So there are $N + 1$ independent variables, i.e., P_{ti} for $i = 1, \ldots, N$ and $T_1 = T_2 = \ldots = T_N$. As will become clearer later on, under PTC-UT, the optimal transmit power at each node can be computed locally based on some common parameters broadcasted by node o. The distributed nature of this scheme reduces the control overhead and simplifies system design. For the PTC-UT scheme, the optimization problem can be expressed as in (5) with the additional constraint:

$$T_1 = T_2 = \ldots = T_N. \tag{7}$$

C. PTC-USG Scheme

For PTC-USG scheme, node o specifies the N transmission powers and the N transmission durations for all nodes in such a way that all N transmissions have the same data rate. Accordingly, there are $N + 1$ independent control variables, i.e., P_{ti} for $i = 1, \ldots, N$ and $R \overset{\text{def}}{=} \frac{B_1}{T_1} = \frac{B_2}{T_2} = \ldots = \frac{B_N}{T_N}$. Similar to PTC-UT, PTC-USG can also be implemented in a distributed fashion. In addition, by taking advantage of the common spreading gain $(\frac{W}{R})$ across different nodes, the implementation can be further simplified by assigning in each cycle the same family of spreading codes for all the sensors. For the PTC-USG scheme, the optimization has the same form as (5) with the additional constraint:

$$\frac{B_1}{T_1} = \frac{B_2}{T_2} = \ldots = \frac{B_N}{T_N}. \tag{8}$$

IV. Analytical Solutions

With some algebraic manipulations [6], it can be shown that the constraints in (5) can be put in the forms of posynomials in $\mathbf{P}_t$ and $\mathbf{T}$, so that the resulting optimization problem is a standard geometric program (GP). Efficient numerical algorithms for solving GPs, e.g., interior point algorithm, are readily available.

Rather than relying on a numerical approach, we concentrate on an approximate analytical solution to the problem. This is obtained by decoupling the joint power/time optimization problem into two sequential sub-problems. The first is a parametric linear optimization on the transmission power with the transmission time $\mathbf{T}$ being the parameter. Then, the optimization on $\mathbf{T}$ is approximately formulated as a convex problem, whose solution is derived either through sequential algorithm (for PTC-IPT) or in closed form (for PTC-UT and PTC-USG). The analytical solution is elaborated as follows.

A. Sub-Problem 1: Parametric Solution of Optimal Transmission Powers

Because the formulations for PTC-UT and PTC-USG can be derived from that of PTC-IPT with an additional constraint on $\mathbf{T}$, we first consider the variable-decoupling of (5).

Treating the transmission time vector $\mathbf{T}$ as a given system parameter with $T_i \leq T_i^{limit}$, (5) is equivalent to the following linear programming problem:

$$\begin{cases} \text{minimize}_{\{P_{t1},\ldots,P_{tN}\}} \sum_{i=1}^{N} P_{ti}T_i \\ s.t. \\ \left(1+\frac{\delta B_i\gamma_i}{WT_i}\right) h_i P_{ti} - \frac{\delta B_i\gamma_i}{WT_i}\sum_{j=1}^{N} h_j P_{tj} \ge \frac{B_i\gamma_i N_0}{T_i}, \\ \qquad\qquad\qquad\qquad\qquad\qquad i=1,\ldots,N \\ P_{ti} \le P_{\max}, \quad i=1,\ldots,N. \end{cases} \tag{9}$$

In [6], we have derived the parametric optimal solution to (9) in terms of the transmission time $\mathbf{T}$

$$P_{ti} = \frac{\delta^{-1}h_i^{-1}g_i}{1-g_\Sigma}, \quad i=1,\ldots,N \tag{10}$$

where P_{ti} has been normalized with respect to the energy of background AWGN, $g_\Sigma \stackrel{\text{def}}{=} \sum_{i=1}^{N} g_i$, and g_i is the *power index* of node i:

$$g_i \stackrel{\text{def}}{=} \frac{\delta B_i\gamma_i}{WT_i+\delta B_i\gamma_i}. \tag{11}$$

Because we have not specified any additional constraints on T_i's in our efforts above, the parametric treatment of (9) and the result in (10) also apply to the formulations for PTC-UT and PTC-USG. Accounting for the maximum transmit power constraint, a necessary condition for the existence of the optimal solution is given by

$$g_i \le \delta h_i P_{\max}, \quad i=1,\ldots,N. \tag{12}$$

and

$$g_\Sigma \le \frac{\delta P_{\max} h_\Sigma}{1+\delta P_{\max} h_\Sigma} < 1 \tag{13}$$

where $h_\Sigma \stackrel{\text{def}}{=} \sum_{i=1}^{N} h_i$.

B. Sub-Problem 2: Optimization of Transmission Times

From (11), it is clear that for given system parameters B_i, γ_i, W, and δ, the power index g_i and the transmission time T_i are equivalent measures in the sense that there is a one-to-one mapping between g_i and T_i:

$$T_i = \frac{\delta B_i\gamma_i}{Wg_i}(1-g_i). \tag{14}$$

In the following, it is more convenient to work with g_i. Let $\mathbf{g} \stackrel{\text{def}}{=} (g_1,\ldots,g_N)$. The problem of determining the optimal value of $\mathbf{g}$ is now considered.

1) PTC-IPT Scheme: In [6], we have shown that the problem of determining the optimal value of $\mathbf{g}$ can be approximately formulated as the following convex problem

$$\begin{cases} \text{minimize}_{\{\mathbf{g}\}} \frac{K}{1-g_\Sigma} + \sum_{i=1}^{N} \frac{\alpha_{ciri}A_i}{g_i} - \sum_{i=1}^{N} \alpha_{ciri}A_i \\ s.t. \\ \frac{\delta B_i\gamma_i}{\delta B_i\gamma_i+WT_i^{limit}} \le g_i \le \delta h_i P_{\max}, \quad i=1,\ldots,N \\ \sum_{i=1}^{n} g_i \le \frac{\delta P_{\max}h_\Sigma}{1+\delta P_{\max}h_\Sigma}. \end{cases} \tag{15}$$

We proved in [6] that the optimal solution to (15) can be derived by first solving the un-bounded optimization problem where the upper and lower bounds on g_i is not imposed, and then sequentially fixing those variables that exceed their bounds. In particular, when the traffic load is reasonably smaller than the network's capacity, the optimal transmission time is located within the polyhedron depicted by the constraints of (15). In this case, the optimal solution is given by

$$g_i^o = \frac{\sqrt{\alpha_{ciri}A_i}}{\sqrt{K}+\sum_{i=1}^{N}\sqrt{\alpha_{ciri}A_i}}, \quad i=1,\ldots,N. \tag{16}$$

Having determined g_i^o, the optimal transmit power and transmission time are derived by substituting (16) into (10) and (14), respectively:

$$P_{ti}^{o(PTC-IPT)} = \frac{\delta^{-1}h_i^{-1}g_i^o}{1-g_\Sigma^o}, \tag{17}$$

$$T_i^{o(PTC-IPT)} = \frac{\delta B_i\gamma_i}{Wg_i^o}(1-g_i^o), \quad i=1,\ldots,N, \tag{18}$$

where $g_\Sigma^o \stackrel{\text{def}}{=} \sum_{i=1}^{N} g_i^o$.

2) PTC-UT Scheme: According to (11) and (7), we have

$$T_i \approx \frac{\delta B_1\gamma_1}{Wg_1}, \tag{19}$$

$$g_i = \frac{B_i\gamma_i}{B_1\gamma_1}g_1, \quad i=1,\ldots,N. \tag{20}$$

Substituting (19), (20), (10) and the constraints (12) and (13) into (5), the problem of determining the optimal transmission time under PTC-UT can be formulated as

$$\begin{cases} \text{minimize}_{\{g_1\}} f(g_1) \stackrel{\text{def}}{=} \frac{C}{1-Dg_1} + \frac{E}{g_1} \\ s.t. \\ g_1^{low} \le g_1 \le g_1^{upp}. \end{cases} \tag{21}$$

where $C \stackrel{\text{def}}{=} \frac{1}{W}\sum_{i=1}^{N} h_i^{-1}B_i\gamma_i$, $D \stackrel{\text{def}}{=} \frac{\sum_{i=1}^{N} B_i\gamma_i}{B_1\gamma_1}$, $E \stackrel{\text{def}}{=} \frac{\delta B_1\gamma_1\sum_{i=1}^{N}\alpha_{ciri}}{W}$, $g_1^{low} \stackrel{\text{def}}{=} \max_i\left\{\frac{\delta B_1\gamma_1}{\delta B_i\gamma_i+WT_i^{limit}}\right\}$, and $g_1^{upp} \stackrel{\text{def}}{=} \min\left\{\frac{\delta P_{\max}h_\Sigma B_1\gamma_1}{(1+\delta P_{\max}h_\Sigma)\sum_{j=1}^{N}B_j\gamma_j}, \min_i\left\{\frac{\delta h_i P_{\max}B_1\gamma_1}{B_i\gamma_i}\right\}\right\}$ are system-defined constants. Note that once the optimal g_1 is found, the optimal g_i, $i=2,\ldots,N$, can be computed from (20).

By taking the second-order derivative of $f(g_1)$ in (21), we can prove that $f(g_1)$ is strictly convex and must have only one unconstrained minimum solution, which is given by solving the following equation for g_1

$$f'(g_1) = \frac{CD}{(1-g_1D)^2} - \frac{E}{g_1^2} = 0. \tag{22}$$

Solving (22), the unconstrained minimum solution is given by

$$g_{u1}^o = \frac{\sqrt{E}}{\sqrt{CD}+\sqrt{E}D} \tag{23}$$

Accounting for the upper and lower bounds given in (21), the constrained optimal solution to (21) is given by

$$g_1^o = \begin{cases} g_{u1}^o, & g_1^{low} \le g_{u1}^o \le g_1^{upp}, \\ g_1^{low}, & g_{u1}^o < g_1^{low}, \\ g_1^{upp}, & g_{u1}^o > g_1^{upp}. \end{cases} \tag{24}$$

Having determined g_1^o, the optimal transmission times and powers are given by substituting g_1^o into (19) and (10), resulting in

$$T_i^{o(PTC-UT)} = \frac{\delta B_1\gamma_1}{Wg_1^o}, \quad i=1,\ldots,N, \tag{25}$$

$$P_{ti}^{o(PTC-UT)} = \frac{\delta^{-1}h_i^{-1}B_i\gamma_i g_1^o}{B_1\gamma_1 - g_1^o\sum_{j=1}^{N}B_j\gamma_j}. \tag{26}$$

3) PTC-USG Scheme: The value of g_i can be approximately presented in terms of R as

$$g_i \approx \frac{\delta\gamma_i R}{W}, \quad i = 1, \ldots, N. \tag{27}$$

Substituting $T_i = \frac{B_i}{R}$, (27), (10) and the constraints (12) and (13) into (5), the problem of determining the optimal R under PTC-USG scheme is formulated as

$$\begin{cases} \text{minimize}_{\{R\}} l(R) \overset{\text{def}}{=} \frac{F}{1-RG} + \frac{H}{R} \\ s.t. \\ R^{low} \le R \le R^{upp}. \end{cases} \tag{28}$$

where $F \overset{\text{def}}{=} \frac{\sum_{i=1}^{N} h_i^{-1}\gamma_i B_i}{W}$, $G \overset{\text{def}}{=} \frac{\delta\gamma_\Sigma}{W}$, $H \overset{\text{def}}{=} \sum_{i=1}^{N} \alpha_{ciri} B_i$, $\gamma_\Sigma \overset{\text{def}}{=} \sum_{i=1}^{N} \gamma_i$, $R^{low} \overset{\text{def}}{=} \max_i \left\{ \frac{WB_i}{\delta B_i \gamma_i + WT_i^{limit}} \right\}$, and $R^{upp} \overset{\text{def}}{=} \min\left\{ \frac{P_{\max} h_\Sigma W}{(1+\delta P_{\max} h_\Sigma)\gamma_\Sigma}, \min_i \left\{ \frac{W h_i P_{\max}}{\gamma_i} \right\} \right\}$ are system-defined constants.

An observation of the objective function $l(R)$ in (28) shows that it has the same form as $f(g_1)$ in (21). Therefore, $l(R)$ must be strictly convex and has only one unconstrained optimal solution

$$R_u^o = \frac{\sqrt{H}}{\sqrt{FG} + \sqrt{HG}}. \tag{29}$$

Accounting for the upper and lower bounds in (28), the constrained optimal solution to (28) is given by

$$R^o = \begin{cases} R_u^o, & R^{low} \le R_u^o \le R^{upp}, \\ R^{low}, & R_u^o < R^{low}, \\ R^{upp}, & R_u^o > R^{upp}. \end{cases} \tag{30}$$

Substituting R^o and (27) into (10), the optimal transmission power and transmission time under PTC-USG scheme are given by

$$T_i^{o(PTC-USG)} = \frac{B_i}{R^o}, \quad i = 1, \ldots, N, \tag{31}$$

$$P_{ti}^{o(PTC-USG)} = \frac{h_i^{-1}\gamma_i R^o}{W - R^o \delta\gamma_\Sigma}, \quad i = 1, \ldots, N. \tag{32}$$

V. Performance Comparison

A. Energy Efficiency

Based on the expressions for the optimal transmission powers and times derived in Section IV, the total energy consumption in a transmission cycle can be studied analytically.

1) PTC-IPT Scheme: From (16), the optimal power index of node i under PTC-IPT scheme is given by

$$\begin{aligned} g_i^{o(PTC-IPT)} &= \frac{\sqrt{\alpha_{ciri} A_i}}{\sqrt{K} + \sum_{j=1}^{N} \sqrt{\alpha_{cirj} A_j}} \\ &= \frac{\sqrt{\alpha_{ciri}\delta B_i \gamma_i}}{\sqrt{\sum_{j=1}^{N} h_j^{-1} B_j \gamma_j} + \sum_{j=1}^{N} \sqrt{\alpha_{cirj} B_j \delta\gamma_j}} \end{aligned} \tag{33}$$

Substituting (33), (17) and (18) into (4), and after some mathematical effort, the total energy consumption in a transmission cycle is given by

$$E_{total}^{(PTC-IPT)} = \frac{1}{\eta W} \left(\sqrt{\sum_{i=1}^{N} h_i^{-1} B_i \gamma_i} + \sum_{i=1}^{N} \sqrt{\alpha_{ciri} B_i \delta\gamma_i} \right)^2. \tag{34}$$

2) PTC-UT Scheme: From (23), the optimal power index g_1^o under PTC-UT scheme is given by

$$\begin{aligned} g_1^{o(PTC-UT)} &= \frac{\sqrt{E}}{\sqrt{CD} + \sqrt{ED}} \\ &= \frac{B_1 \gamma_1 \sqrt{\delta \sum_{i=1}^{N} \alpha_{ciri}}}{\sqrt{\sum_{i=1}^{N} h_i^{-1} B_i \gamma_i \sum_{i=1}^{N} B_i \gamma_i} + \sqrt{\delta \sum_{i=1}^{N} \alpha_{ciri}} \sum_{i=1}^{N} B_i \gamma_i} \end{aligned} \tag{35}$$

Substituting (35) into (25), (26), and (4), and following a similar mathematical manipulation to the one used for the PTC-IPT scheme, the total energy consumption in a transmission cycle under PTC-UT is given by

$$E_{total}^{(PTC-UT)} = \frac{1}{\eta W} \left(\sqrt{\sum_{i=1}^{N} h_i^{-1} B_i \gamma_i} + \sqrt{\delta \sum_{i=1}^{N} \alpha_{ciri}} \sqrt{\sum_{i=1}^{N} B_i \gamma_i} \right)^2. \tag{36}$$

3) PTC-USG Scheme: Similar to the process of PTC-UT, the total energy consumption in a transmission cycle under PTC-USG scheme can be derived as

$$E_{total}^{(PTC-USG)} = \frac{1}{\eta W} \left(\sqrt{\sum_{i=1}^{N} h_i^{-1} B_i \gamma_i} + \sqrt{\sum_{i=1}^{N} \alpha_{ciri} B_i} \sqrt{\delta \sum_{i=1}^{N} \gamma_i} \right)^2. \tag{37}$$

B. Comparison of Energy Consumption

In order to compare the energy efficiency of different schemes, we assume for simplicity a homogeneous WSN, i.e., $\alpha_{ciri} = \alpha_{cir}$ and $\gamma_i = \gamma$ for all i. With this assumption, the energy consumption under various schemes can be further simplified as

$$E_{total}^{(PTC-IPT)} = \frac{1}{\eta W} \left(\sqrt{\sum_{i=1}^{N} h_i^{-1} B_i \gamma} + \sqrt{\alpha_{cir}\delta\gamma} \sum_{i=1}^{N} \sqrt{B_i} \right)^2 \tag{38}$$

and

$$\begin{aligned} E_{total}^{(PTC-UT)} &= E_{total}^{(PTC-USG)} \\ &= \frac{1}{\eta W} \left(\sqrt{\sum_{i=1}^{N} h_i^{-1} B_i \gamma} + \sqrt{\alpha_{cir}\delta\gamma N} \sqrt{\sum_{i=1}^{N} B_i} \right)^2. \end{aligned} \tag{39}$$

The only difference between (38) and (39) is in the second component in the base. Because $\sqrt{x}$ is a concave function, according to Jensen's inequality, we have

$$\sqrt{\frac{1}{N} \sum_{i=1}^{N} B_i} \ge \frac{1}{N} \sum_{i=1}^{N} \sqrt{B_i} \tag{40}$$

or equivalently $\sqrt{N}\sqrt{\sum_{i=1}^{N} B_i} \ge \sum_{i=1}^{N} \sqrt{B_i}$. Therefore, $E_{total}^{PTC-UT} = E_{total}^{PTC-USG} \ge E_{total}^{(PTC-IPT)}$, which is in line with the intuition that PTC-IPT should be more energy-efficient because of its larger degree of control.

node	h_i ($\times 10^{-6}$)	B_i (bits)	PTC-IPT (GP)		PTC-IPT		PTC-UT		PTC-USG		MDT	
			P_{ti}^o	T_i^o	P_{ti}^o	T_i^o	P_{ti}^o	T_i^o	P_{ti}^o	T_i^o	P_{ti}^o	T_i^o
1	0.0634	98	2.319	3.8	2.315	3.8	2.224	4.2	2.410	3.9	0.00619	1000
2	0.0068	124	24.326	4.2	24.113	4.2	26.056	4.2	22.315	4.9	0.0725	1000
3	0.029	86	4.75	3.5	4.743	3.5	4.269	4.2	5.271	3.4	0.01188	1000
4	0.8816	112	0.178	4.0	0.178	4.0	0.183	4.2	0.173	4.4	5.1×10^{-4}	1000
5	0.0029	111	53.967	4.0	54.327	4.0	55.541	4.2	53.140	4.4	0.1545	1000
E_{total}^o	-	-	576.49		577.67		621.31		621.31		5.027×10^4	

TABLE I

PARAMETERS AND OPTIMIZATION RESULTS FOR A 5-NODE CDMA WSN, WHERE THE UNITS OF POWER, TIME, AND ENERGY ARE mW, ms, AND μJ, RESPECTIVELY ($\alpha_{cir} = 10$mW, $P_{\max} = 100$mW).

C. Comparison of Computation Overhead

PTC-UT and PTC-USG are easier to implement than PTC-IPT. The number of control variables in PTC-UT and PTC-USG is approximately half of that in PTC-IPT. In addition, as shown in [6], under PTC-IPT a sequential algorithm need to be executed at node o to solve for the optimal transmission powers and times. In contrast to this centralized operation, the optimization under PTC-UT and PTC-USG can be realized distributedly at each node with some assistance from node o. Specifically, given the values of g_1^o, $B_1\gamma_1$, and $\sum_{i=1}^N B_i\gamma_i$, the optimal transmit power and time under the PTC-UT scheme can be computed locally by each node according to (25) and (26). Because the required information is the same for all nodes, node o simply needs to broadcast them throughout the system. Similarly, (31) and (32) show that under the PTC-USG scheme, broadcasting the values of R^o and $\sum_{i=1}^N \gamma_i$ is sufficient for local computation of the optimal transmission power and time at individual nodes. As a result, PTC-UT and PTC-USG have much smaller computing complexity than PTC-IPT.

VI. NUMERICAL EXAMPLES

We consider a cluster consisting of 5 homogeneous sensor nodes and one cluster head. For each sensor node, the energy efficiency of the power amplifier is $\eta = 0.9$. We assume the orthogonality factor $\delta = \frac{2}{3}$. The threshold of the received SINR is 4. Each transmission must be completed within $T_i^{limit} = 1$ second. The spread spectrum bandwidth is $W = 1$ MHz and $N_0 = 10^{-15}$ W/Hz. The channel and traffic parameters of each node and the optimization results are listed in Table 1.

In Table 1, the *maximum delay transmission* (MDT) scheme is included. This scheme always assigns the longest possible transmission time (i.e., the largest delay) to each node and calculates the optimal transmission power by using (10). When circuit energy consumption is ignored, MDT is the optimal control scheme that minimizes the total transmission energy [6]. However, in a WSN where the circuit energy is non-negligible, Table 1 shows that significant energy savings over MDT can be achieved by jointly optimizing the transmission power and time. It is further noted that while PTC-IPT involves nearly twice as many control variables as PTC-UT and PTC-USG, it achieves only minor efficiency improvement (around 7.6%). This observation is justified by comparing the power consumption equations of PTC-IPT, PTC-UT, and PTC-USG scheme, as given by (38) and (39). More specifically, because of the impact from the channel gain, the energy consumption under these schemes is dominated by the first component in the base in (38) and (39), which is the same for both equations. At the same time, as illustrated in (40), the second component in (39) is actually a tight upper bound than the one in (38), leading to only a slight difference in energy efficiency.

To examine the accuracy of the closed-form solutions derived in the previous sections, we include in Table 1 the results computed from the GP-based numerical algorithm. It can be observed that there is almost no difference between the closed-form result and those from the GP-based numerical algorithm.

VII. CONCLUSIONS

In this paper, we investigated the energy efficiency and control complexity of a CDMA WSN under three different joint power/time control schemes: PTC-IPT, PTC-UT, and PTC-USG. We found that although PTC-IPT has complete freedom in controlling both transmission power and time for every sensor node, it brings minor improvement in the energy efficiency compared with its simplified versions (PTC-UT and PTC-USG). This indicates that energy efficiency is not sensitive to the difference in transmission time of individual sensor nodes, thus there may be no need to separately control the transmission time of each node.

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