Distributed Spectrum Management in TV White Space Networks

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***Abstract*—We investigate the distributed spectrum management problem in opportunistic TV White Space (TVWS) systems using a game theoretical approach that accounts for adjacent-channel interference and spatial reuse. TV Bands Devices (TVBDs) compete to access idle TV channels and select channel "blocks" that optimize an objective function. This function provides a tradeoff between the achieved rate and a cost factor that depends on the interference between TVBDs. We consider practical cases where contiguous or non-contiguous channels can be accessed by TVBDs, imposing realistic constraints on the maximum frequency span between the aggregated/bonded channels. We show that under general conditions, the proposed TVWS management games admit a potential function. Accordingly, a "best response" strategy allows us to determine the spectrum assignment of all players. This algorithm is shown to converge in a few iterations to a Nash Equilibrium (NE). Furthermore, we propose an effective algorithm based on Imitation dynamics, where a TVBD probabilistically imitates successful selection strategies of other TVBDs in order to improve its objective function. Numerical results show that our game theoretical framework provides a very effective tradeoff (close to optimal, centralized spectrum allocations) between efficient TV spectrum use and reduction of interference between TVBDs.**

***Index Terms*—TV White Space, Spectrum Management, Channel Bonding/Aggregation, Game Theory, Nash Equilibrium.**

I. Introduction

The radio frequency (RF) spectrum is a scarce resource that has recently become particularly critical with the increased wireless demand [1]. For this reason, the Federal Communications Commission (FCC) has recently allowed for opportunistic access to the unused spectrum in the TV bands (also called "white space"). With opportunistic access, however, there is a need to deploy enhanced channel allocation and power control techniques [2], [3], [4], [5], [6] to mitigate interference, including Adjacent-Channel Interference (ACI) [7], [8]. TV White Space (TVWS) spectrum access is often investigated without taking into account ACI between the transmissions of TV Bands Devices (TVBDs) and licensed TV stations. Guard Bands (GBs) can be used to protect data transmissions and mitigate the ACI problem.

In this work, we consider a spectrum database that is administrated by a Database Operator (DO), and an opportunistic secondary system, in which every TVBD is equipped with a single antenna that can be tuned to a subset of licensed channels. This can be done, for example, through adaptive channel aggregation or bonding techniques [9], [10], [11]. We take into account practical constraints on hardware and aggregation overhead, which are translated into a maximum allowable span (d_{max}) between the two farthest channels chosen for aggregation by the TVBD. Furthermore, we accurately model spatial reuse and interference among TVBDs, and we consider both static and dynamic scenarios, where TVBDs move around in their environment.

Given the set of channels occupied by licensed TV stations, we assume that the DO first adds GBs to protect ongoing TV transmissions, and then provides the set of idle channels and guard bands to unlicensed TVBDs. According to the FCC specifications [12], [13], each TVBD must contact the DO to obtain the list of idle TV channels and then decide which ones to use in order to maximize its own performance (which can be expressed as a function of interference/congestion). If multiple TVBDs are located in the same area, they will receive the same channel occupancy information, and hence they will likely interfere with each other. Game theory is a natural framework to address the conflicts between such self-interested devices (or players), and the Nash Equilibrium (NE) is a well-suited concept to characterize the system-wide equilibrium conditions.

In [14], the authors conducted a game theoretic analysis of a distributed spectrum sharing scheme with a geo-location database. They modeled the channel selection problem among Access Points (APs) as a distributed channel selection game and proposed a state-based game framework to model the distributed association of secondary users to APs, taking into account the cost of mobility. Two pricing schemes (registration and service plans) for TVWS database were proposed in [15]. The DO offers the two schemes in order to maximize its benefit (the payment received from all Secondary Users, SUs). Then, SUs access idle TV channels so as to maximize their utility, expressed as a function of the Shannon capacity. The competition among SUs is modeled as a non-cooperative game under both complete and incomplete information. In [16], the authors investigated an oligopoly competitive TV white space market, where multiple secondary network operators compete to serve a common pool of secondary end-users by using TV white space purchased from a spectrum broker.

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In [17], the authors proposed a non-cooperative TVWS spectrum management game, where TVBDs choose a number of idle blocks to optimize their objective function, expressed in terms of a price set by the DO and a congestion-based cost. However, in some cases the per-block channel assignment can be very inefficient. For this reason, in this work we study and formulate the TVWS spectrum management games on a *per channel* basis.

Recently, optimal GB-aware (GBA) channel assignment schemes were proposed in [18] and [8] for multi-channel dynamic spectrum access networks. The authors in [18] formulated and obtained optimal GBA channel assignment for a single and multiple links, and they showed that the proposed GBA channel assignment scheme achieves optimal spectrum efficiency and supports channel bonding and aggregation. However, the transmission rate was assumed to be deterministically known. In [8], the authors considered a more general case, where channel quality is uncertain, and to account for this uncertainty, they developed stochastic GBA channel assignment schemes.

Various recent works use auction design and apply game theory to incentivize users to share their spectrum (e.g., [19]), but they do not consider channel bonding as we do in our work. Our key contributions to the TVWS spectrum management problem are as follows: 1) we consider a general utility function that adequately captures the interference among TVBDs (guaranteeing a minimum data rate requirement), and 2) we incorporate channel bonding in our analysis, which is an important consideration for supporting high-rate demands in opportunistic TV whitespaces. This issue has not been addressed in existing game theoretical analyses.

The main contributions of this paper can be summarized as follows:

- We tackle the TV spectrum management problem both in a fully distributed as well as centralized settings, capturing various practical constraints. In particular, we model the ACI and spatial reuse, and we assume that each TVBD can choose a set of idle channels (not necessarily contiguous) by implementing bonding and/or aggregation techniques.
- We formulate the distributed TV spectrum management problem using two non-cooperative spectrum management games. In the first game, TVBDs can choose a set of *non-contiguous* idle channels while satisfying some hardware constraints (e.g., maximum frequency span). In the second game, TVBDs can only choose *contiguous* channels. In the two games, the TVBD attempts to optimize its objective function.
- Under certain conditions, detailed in Section III, we demonstrate that the non-cooperative games played by TVBDs are exactly potential, and hence admit at least one pure-strategy NE. We also show that the Best Response algorithm converges in few iterations to a NE.
- We further propose an *Imitation-based* algorithm for our games, where a TVBD can imitate with a certain probability other (successful) TVBDs while selecting its strategy, in order to improve its objective function.
- We compare our games to the centralized (Social Welfare) solutions, which provide bounds on the Price of Anarchy (PoA) [20]. Considering both static (with fixed transmission power) and dynamic TVWS scenarios (where users are mobile), we perform extensive numerical analysis and show that our games and solution approaches always provide a very efficient solution for managing TV resources in a distributed manner.

The remainder of this paper is organized as follows. Section II introduces the system model as well as the notation and assumptions made in the paper. Section III describes the game theoretic approaches for the TV spectrum management problem, and demonstrates that under certain conditions our games admit a potential function. Section IV presents the two proposed distributed TVWS spectrum management algorithms: the Best Response and the Imitation-based algorithms. Numerical results are provided in Section V. Finally, concluding remarks and future research are discussed in Section VI.

II. System Model

In this section, we describe the system model and the notation used throughout the paper. We consider the TVWS scenario in Figure 1, where a spectrum database is administrated by a third party DO. The DO serves a set $\mathcal{N}$ of unlicensed TVBDs. Potentially available TV channels include channels 2 to 51 (except channels 3, 4 and 37) in the case of fixed TVBDs, or channels 21 to 51 (except channel 37) for personal/portable TVBDs [12], [13].

Following the FCC's 3rd MO&O [13], we remark that *"fixed devices may operate only on vacant TV channels that are not adjacent to occupied TV channels, while personal/portable devices may operate adjacent to occupied channels if their maximum EIRP is reduced to no more than 40 mWatt (instead of 100 mWatt EIRP)"*. Furthermore, TVBDs must incorporate a geo-location capability and a means to access the database to retrieve a list of idle TV channels that may be used at a given location [12], [13]. They use a fixed transmission power, i.e., power control is not applied. TVBDs may also perform spectrum sensing to determine the relative utilization of a given channel. Therefore, in the rest of the paper, we assume that the DO first provides all TVBDs with the set of idle, guard band, and occupied channels. Based on such information, each TVBD i chooses at most n_{max} idle channels so as to optimize its objective function. Note that if the TVBD chooses non-contiguous idle channels, it is necessary to guarantee that the distance between the chosen channels does not exceed a given value d_{max}, determined by hardware constraints and aggregation overhead.

We assume that TVBDs are located in the same geographical area, and therefore they perceive the same TV spectrum status. Let $\mathcal{M}$ denote the set of idle TV channels and B the TV channel bandwidth in MHz (same for all channels). Figure 2 illustrates an example. We classify channels into idle, busy, and guard band channels. In Figure 2, channels $\{8, 10, 16, 17\}$ are busy and channels $\{7, 9, 11, 15, 18\}$ are guard bands. Thus, $\mathcal{M} = \{5, 6, 12, 13, 14, 19, 20, 21, 22\}$.

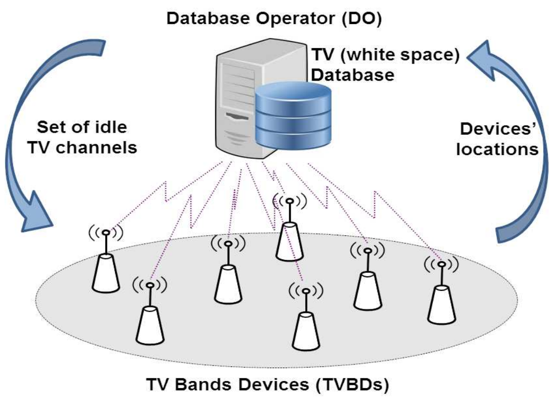


Fig. 1: A TV White Space scenario composed of a set of TVBDs and a TV/white space Database operated by a third-party Database Operator (DO). The DO receives the TVBDs' locations and then provides them with the set of idle TV channels.

Let d_i be the rate demand (in Mbps) required by TVBD i, and let r_j be the maximum data rate that can be supported by TV channel j. In fact, r_j can be deterministic (ideal channel quality) or random if we assume that channel quality varies due to multi-path fading and shadowing, as well as due to the unpredictability of TVBDs activities. Note that under poor channel conditions, we expect that our games will allocate more channels to TVBDs to guarantee their minimum rate demands.

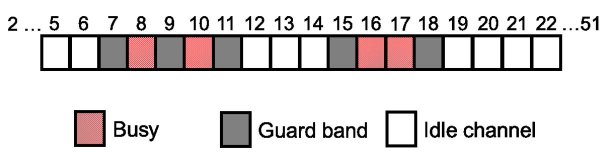


Fig. 2: Example illustrating idle, busy, and guard band channels in the TV spectrum. The set of idle TV channels is given as $\mathcal{M} = \{5, 6, 12, 13, 14, 19, 20, 21, 22\}$.

III. Game theoretic TVWS spectrum management

We address the TVWS spectrum management problem in a fully distributed fashion using a game theoretic approach. Given the spectrum status that is provided by the DO, each TVBD locally selects a set of idle channels (at most n_{max}) for its communications so as to optimize an objective function (III-A), which is related to both the utility perceived by using the chosen channels and a cost term, expressed as a function of the experienced interference.

We study and compare two variants of the TVWS spectrum management game:

- Game $\mathcal{G}_1$ (Channel Aggregation-based TVWS Spectrum Management, III-B): Given the spectrum status, TVBDs play the game choosing *at most n_{max} idle channels*, which are *not necessarily contiguous*. However, the chosen idle channels must be separated by no more than d_{max} channels. This feature is used to take into account hardware constraints and the cost of aggregating distant channels.
- Game $\mathcal{G}_2$ (Channel Bonding-based TVWS Spectrum Management, III-C): Given the spectrum status, TVBDs play the game choosing *at most n_{max} contiguous idle channels*. This condition is used to minimize the system complexity (aggregation overhead, hardware costs), guaranteeing a fair access to TVWS, independent of rate demands.

Hereafter, we elaborate on $\mathcal{G}_1$ and $\mathcal{G}_2$, considering the example in Figure 2. We focus on channels 5 to 18, for simplicity. According to $\mathcal{G}_1$, if the TVBD can choose at most $n_{max} = 2$ idle channels separated by a distance of at most $d_{max} = 6$, then its possible choices are: $\{5, 6\}$, $\{6, 12\}$, $\{12, 13\}$, $\{12, 14\}$, and $\{13, 14\}$. On the other hand, in $\mathcal{G}_2$, if the TVBD can choose at most $n_{max} = 2$ contiguous idle channels, then it has the following alternatives: $\{5, 6\}$, $\{12, 13\}$, and $\{13, 14\}$, besides choosing each of these channels separately. Of course, the strategy space of the TVBD in $\mathcal{G}_1$ is in general larger than that of $\mathcal{G}_2$.

Finally, we demonstrate that both games $\mathcal{G}_1$ and $\mathcal{G}_2$ exhibit desirable properties since they are potential (see Appendix A), and possess at least one pure-strategy Nash Equilibrium (NE). Hence, a Best Response algorithm can be used to converge to a NE.

TABLE I: Basic Notation

$\mathcal{N}$	Set of TV Bands Devices (TVBDs)
$\mathcal{M}$	Set of idle TV channels
B	Bandwidth of each TV channel
r_j	Rate (Mbps) supported by TV channel j
α_j, β_j, γ_j	Channel j-specific parameters
δ_{ij}	TVBD i-specific parameter on channel j
$e_{ki}^{(j)}$	Interference parameter between TVBDs k and i on channel j
d_i	Minimum rate demand (Mbps) of TVBD i
d_{max}	Maximum distance between chosen channels
n_{max}	Per-TVBD maximum number of channels
x_{ij}	Binary variable that indicates if TVBD i is assigned to idle channel j

The basic notation used throughout the paper is summarized in Table I. We define the binary decision variables $x_{ij} \forall i \in \mathcal{N}$ and $\forall j \in \mathcal{M}$ as follows:

$$x_{ij} = \begin{cases} 1 & \text{if idle channel } j \text{ is assigned to} \\ & \text{TVBD } i\text{'s transmission} \\ 0 & \text{otherwise.} \end{cases}$$

These variables represent the set of spectrum access strategies of TVBD i, i.e., $\boldsymbol{x}_i = \{x_{i1}, x_{i2}, \ldots, x_{i|\mathcal{M}|}\}$.

We denote by E_j the interference matrix associated with idle channel j. Let $e_{ik}^{(j)}$ be the $(i,\ k)$th element of E_j, the interference parameter between TVBDs i and k on channel j. Note that E_j needs not be symmetric. More specifically, $e_{ik}^{(j)}$, for $i, k \in \mathcal{N}$ and $j \in \mathcal{M}$, is defined as follows:

$$e_{ik}^{(j)} = \begin{cases} 1 & \text{if TVBD } i \text{ interferes with TVBD } k \text{ on} \\ & \text{channel } j \\ 0 & \text{otherwise.} \end{cases}$$

A. Objective Function

We now introduce the objective function optimized by each TVBD. We begin by illustrating the *cost function* J_i of TVBD $i, \forall i \in \mathcal{N}$, which represents a congestion cost that the device incurs due to its *interference* on other devices that operate on the same channel j. J_i is given by:

$$J_i = \sum_{j \in \mathcal{M}} r_j x_{ij} \cdot [\alpha_j \cdot (\sum_{k \in \mathcal{N}} r_j e_{ki}^{(j)} x_{kj})^{\beta_j} + \gamma_j]. \quad (1)$$

where the coefficients α_j and γ_j are two positive numbers that model the overhead caused by choosing a wireless channel j, and β_j is a positive integer greater than or equal to 1 (the larger is β_j, the higher is the impact of interference among TVBDs).

The above cost function well captures the network congestion level and it is commonly used in the literature [2], [4]. More specifically, for each channel j we consider an increasing and convex function of the form:

$$\alpha_j \cdot (\sum_{k \in \mathcal{N}} r_j e_{ki}^{(j)} x_{kj})^{\beta_j} + \gamma_j \quad (2)$$

where $r_j x_{kj}$ is the traffic of TVBD k over channel j.

We observe that (2) represents the per traffic unit congestion cost experienced by the TVBD on a single channel. Therefore, the total cost incurred by device i due to the overall network congestion is obtained by summing the cost over all channels.

J_i represents the penalty that TVBD i pays due to interference. It is monotone in the number of TVBDs sharing the same band, and is used to incite them to choose idle or underutilized bands. In other words, this cost is naturally proposed to discourage TVBDs from choosing "crowded" channels, thus reducing the interference. Hence, each TVBD i is better off minimizing J_i.

In this work, we focus on TVBDs characterized by a minimum data rate requirement (d_i) and elastic traffic: the goal of each device is to maximize the difference between its utility (U_i) and cost (J_i). We consider an affine utility function of the form:

$$U_i = \sum_{j \in \mathcal{M}} \delta_{ij} r_j x_{ij} \quad (3)$$

where δ_{ij} is a positive parameter that represents the significance (priority) of channel j for TVBD i. Hence, the objective function that (elastic) TVBD i aims to maximize is given by:

$$\begin{aligned} OF_i =& U_i - J_i \quad (4) \\ =& \sum_{j \in \mathcal{M}} \delta_{ij} r_j x_{ij} - \\ & \sum_{j \in \mathcal{M}} r_j x_{ij} \cdot [\alpha_j \cdot (\sum_{k \in \mathcal{N}} r_j e_{ki}^{(j)} x_{kj})^{\beta_j} + \gamma_j]. \end{aligned}$$

It is worth noting that there is a tradeoff between minimizing the number of chosen idle channels and minimizing the interference with other TVBDs.

Having defined the objective function, we now formalize games $\mathcal{G}_1$ and $\mathcal{G}_2$.

B. Game $\mathcal{G}_1$: Channel Aggregation-based TVWS Spectrum Management

In $\mathcal{G}_1$ each player i aims at maximizing OF_i in (4) subject to the following constraints:

Rate demand constraint:

$$\sum_{j \in \mathcal{M}} r_j x_{ij} \geq d_i \quad (5)$$

Maximum number of channels constraint (at most n_{max} channels can be chosen by a TVBD):

$$\sum_{j \in \mathcal{M}} x_{ij} \leq n_{max} \quad (6)$$

Maximum frequency-separation constraint (which guarantees that the maximum separation between any chosen channels j_1 and j_2 does not exceed d_{max}):

$$j_1 x_{ij_1} - j_2 x_{ij_2} \leq d_{max} + (1 - x_{ij_2}) \cdot |\mathcal{M}|, \quad (7)$$
$$\forall j_1, j_2 \in \mathcal{M} : j_1 > j_2$$

Integrality constraints:

$$x_{ij} \in \{0, 1\}, \forall j \in \mathcal{M} \quad (8)$$

C. Game $\mathcal{G}_2$: Channel Bonding-based TVWS Spectrum Management

In $\mathcal{G}_2$ player i maximizes his objective function OF_i subject to constraints (5), (6), and (8) in $\mathcal{G}_1$. In addition, the following single frequency block constraint (contiguous channels) is imposed:

$$j_1 x_{ij_1} - j_2 x_{ij_2} \leq n_{max} - 1 + (1 - x_{ij_2}) |\mathcal{M}|, \quad (9)$$
$$\forall j_1, j_2 \in \mathcal{M} : j_1 > j_2$$

D. Potential Function and Existence of NE

We now demonstrate that under some conditions to be specified later, games $\mathcal{G}_1$ and $\mathcal{G}_2$ admit a potential function Φ. Indeed, if a potential function exists, these games are *potential* [21], and possess at least one pure-strategy Nash Equilibrium (NE). Hence, a Best Response algorithm can be used to converge to a NE.

Because NE uniqueness in discrete games is generally hard to prove, the BR algorithm performs like a local optimization algorithm and the obtained solutions constitute local maxima.

Proposition III.1. *Games $\mathcal{G}_1$ and $\mathcal{G}_2$ admit a potential function Φ, which is given by the following expression:*

$$\begin{aligned} \Phi =& \sum_{i \in \mathcal{N}} \left[U_i - \frac{1}{2} J_i\right] \quad (10) \\ & - \frac{1}{2} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{M}} \left[\alpha_j r_j^2 x_{ij}^2 + r_j \gamma_j x_{ij}\right]. \end{aligned}$$

Proof: See Appendix A.

IV. Distributed TVWS Spectrum Management Algorithms

In this section, we first present two *best response*-based dynamics for both TVWS spectrum management games $\mathcal{G}_1$ and

$\mathcal{G}_2$: a sequential dynamics, where players iteratively change their strategy based on full information of all TVBDs' choices, and a Krasnoselskij-based scheme; with this latter, we model a realistic situation in which, at each iteration, only a fraction of players change their strategy. We further propose an *imitation algorithm* which can be easily implemented in practice, since it needs (much) less information than the best-response-based schemes.

A. Best Response-based Distributed Spectrum Management (BR-DSM) Algorithm

The best response of a player (or a TVBD) is an action (i.e., a set of idle channels) that maximizes its objective function OF_i for a given action tuple of the other players, subject to constraints (5)-(8) for $\mathcal{G}_1$, and to constraints (5), (6), (8) and (9) for $\mathcal{G}_2$.

Definition IV.1. *BR_i is a best response for player i to x_{-i} if*

$$BR_i = argmax_{x_i \in X_i} OF_i(x_i, x_{-i}), \quad \text{s.t. constraints of } \mathcal{G}_1/\mathcal{G}_2. \tag{11}$$

The same procedure is repeated for all TVBDs in the network, and such procedure converges iteratively to a Nash equilibrium of our games.

A formal description of BR-DSM is illustrated in Algorithm 1. Each player sequentially updates its strategy based on the choices of all other players. The procedure stops when no player can improve its own utility, so that a Nash equilibrium has been reached. We underline that, since we demonstrated that our games are exactly potential, BR-DSM is guaranteed to converge to a NE point.

Algorithm 1: BR-DSM

Input : $\mathcal{N}, \mathcal{M}, \boldsymbol{d_i}, \boldsymbol{r}, B, \boldsymbol{x}_0$
Output: $\boldsymbol{x}_{NE}, \boldsymbol{OF}_{NE}, n_{iter}$
1 **Initialization:**
$\boldsymbol{x}_t = \boldsymbol{x}_0, \boldsymbol{OF}_t = \boldsymbol{OF}(\boldsymbol{x_0}), \boldsymbol{OF}_{t-1} = \boldsymbol{0}, n_{iter} = 0, t = 0;$
2 **while** $\Delta \boldsymbol{OF} \neq \boldsymbol{0}$ *(NE solution is not reached)* **do**
3 $\quad t = t + 1;$
4 $\quad$ **foreach** $i \in \mathcal{N}$ **do**
5 $\quad\quad n_{iter} = n_{iter} + 1;$
6 $\quad\quad BR_i = argmax_{x_i} OF_i(x_{i,t}, x_{-i,t})$, s.t. constraints of $\mathcal{G}_1/\mathcal{G}_2$;
7 $\quad\quad OF_t(i) = OF(BR_i, x_{-i,t});$
8 $\quad\quad x_t = \{BR_i, x_{-i,t}\};$
9 $\quad$ **end**
10 $\quad \Delta \boldsymbol{OF} = \boldsymbol{OF}_t - \boldsymbol{OF}_{t-1};$
11 **end**
12 $\boldsymbol{x}_{NE} = \boldsymbol{x}_t, \boldsymbol{OF}_{NE} = \boldsymbol{OF}_t.$

B. Krasnoselskij-based Distributed Spectrum Management (K-DSM) Algorithm

According to BR-DSM, *all* TVBDs change sequentially their strategies at each iteration of the algorithm when determining the NE solution. Nonetheless, assuming that all devices change simultaneously their strategies is not a realistic representation of the market, as players may obtain side-information and make their decisions at different time instants. Therefore, we propose to use the Krasnoselskij algorithm [22], [23], where only a *fraction* of TVBDs change their strategies at the same time at each iteration to improve their objective function and, eventually, converge to a NE of the TVWS spectrum management game. A formal description of K-DSM can be straightforwardly deduced from Algorithm 1 by considering at step 4 only a subset of TVBDs $\mathcal{N}$ (i.e., $\lambda\%$ of the $|\mathcal{N}|$ players, randomly chosen, who may change their strategies at each iteration). This can be easily implemented in practice in a fully distributed fashion. We do that by letting each TVBD extract a random value (for example, uniformly in $[0, 1]$), and then decide to update its strategy if the extracted value is less than $\lambda/100$. Note that we evaluated numerically the impact of different probability distributions of the random value, but no change in the results was observed since all players, sooner or later, update their strategy (as indeed will happen with the distributed random extraction mechanism we described above). We observed the convergence to the same Nash Equilibrium in all considered scenarios and in all cases.

C. Imitation-based Distributed Spectrum Management (IM-DSM) Algorithm

Finally, we present hereafter a distributed spectrum management algorithm based on *imitation dynamics*, a behavior rule widely applied in human societies consisting of imitating successful behavior. This type of technique has been used, for example, in [24], [25] to tackle the distributed spectrum access problem in the context of cognitive radio networks, and has the appealing property of being easily implementable in our scenario.

Algorithm 2 presents our proposed spectrum management policy based on the proportional imitation rule, named IM-DSM (Imitation-based Distributed Spectrum Management) Algorithm. The basic idea of IM-DSM is as follows:

- At each iteration t, each TVBD (say i) randomly selects another TVBD (say $k \neq i$);
- if the objective function value OF at $t-1$ of the *selected* TVBD ($OF_{t-1,k}$) is higher than its *own* objective function value at $t-1$ ($OF_{t-1,i}$), plus δ, a small constant[1], TVBD i imitates the strategy of k at iteration t with a probability q proportional to the objective function values difference (of i and k), with a coefficient σ representing the imitation factor.

We observe that this algorithm exploits a limited information with respect to the best response dynamics, since it is based exclusively on the knowledge of the utility obtained in the previous step by the chosen player.

We consider different variants of this algorithm. In particular:

[1] This permits to avoid so-called *ping-pong* effects. In this regard, the algorithm converges towards a $\delta - NE$, but since δ is very small, such value is very close to the NE. In such algorithms, σ is also set to a small value to achieve stability.

Algorithm 2: IM-DSM

Input : $\mathcal{N}, \mathcal{M}, \boldsymbol{d_i}, \boldsymbol{r}, B, \boldsymbol{x}_0, \sigma, \delta$
Output: $\boldsymbol{x}_{NE}, \boldsymbol{OF}_{NE}, n_{iter}$
Initialization:
$\boldsymbol{x}_t = \boldsymbol{x}_0, \boldsymbol{OF}_t = \boldsymbol{OF}(\boldsymbol{x_0}), \boldsymbol{OF}_{t-1} = \boldsymbol{0}, n_{iter} = 0, t = 0;$
while $\Delta \boldsymbol{OF} \neq \boldsymbol{0}$ *(NE solution is not reached)* **do**
  $t = t + 1;$
  foreach $i \in \mathcal{N}$ **do**
    $n_{iter} = n_{iter} + 1;$
    Randomly select a TVBD k ($k \neq i$);
    if $OF_{t-1,i} - \delta < OF_{t-1,k}$ **then**
      Migrate to the n_{max} idle channels chosen by TVBD k at iteration $t-1$ ($x_{i,t} = x_{k,t-1}$) with probability $q = \sigma(OF_{t-1,k} - OF_{t-1,i})$;
      $OF_t(i) = OF(x_{i,t}, x_{-i,t-1});$
    end
  end
  $\Delta \boldsymbol{OF} = \boldsymbol{OF}_t - \boldsymbol{OF}_{t-1};$
end
$\boldsymbol{x}_{NE} = \boldsymbol{x}_t, \boldsymbol{OF}_{NE} = \boldsymbol{OF}_t.$

- **i)** TVBD i can choose the player to imitate (k) only among the set of those that were transmitting at time $t-1$ on the same channel (so that only local interactions can be exploited among TVBDs);
- **ii)** the DO broadcasts, at the end of each epoch (say, at the end of slot $t-1$) the occupancy on each channel $c \in \mathcal{M}$, so that each player can indeed switch (probabilistically, and proportionally to the utility gain, as explained before) to the idle channels that guaranteed the maximum utility in the previous iteration.

V. Numerical Results

In this section, we measure the sensitivity of our game theoretic approaches to different key parameters that characterize TVWS systems, like the number of TVBDs and idle TV channels, the interference between TVBDs as well as the rate demands, and finally of parameters d_{max} and λ, in several network scenarios.

We first describe the simulation setup (Sec. V-A), and then we analyze and discuss numerical results (Sec. V-B), focusing on both static (Sec. V-B1) and dynamic scenarios (Sec. V-B2); finally, we characterize the efficiency of the equilibrium achieved in our proposed games through the determination of the Price of Anarchy (Sec. V-B3).

The performance metrics we consider are (1) the TVBD objective function ($OF_i = U_i - J_i$) for both games $\mathcal{G}_1$ and $\mathcal{G}_2$, and (2) the Price of Anarchy (PoA), which is defined in our context as the ratio between the utility of the socially optimal solution and that of the worst Nash equilibrium [20]. The solutions of the proposed games are computed considering several state-of-the-art algorithms. These algorithms include: (1) the classic Best Response algorithm in its sequential and distributed version used in several papers like [2], [14], [16], (2) the Krasnoselskij-based algorithm, which was adopted in [23] as well as other works, and (3) the imitation algorithm used in [24], [25].

A. Simulation Setup

In our simulations, we consider a TV white space system composed of M TV channels and N TVBDs randomly scattered over a 1500 meter × 1500 meter area. The bandwidth of each TV channel is 6 MHz, and the rate r_j supported by TV channel j can be either deterministic or vary according to a random distribution.

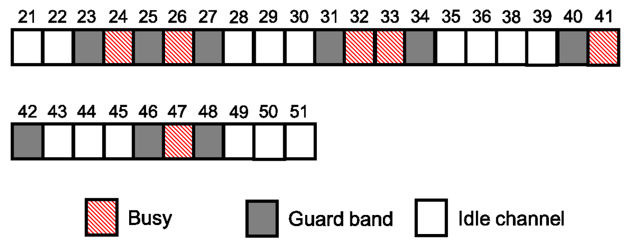


Fig. 3: Set of TV channels ($\{21, \ldots, 51\} \setminus \{37\}$) considered in the numerical analysis.

We simulate both static and dynamic TVWS scenarios. In static scenarios TVBDs are fixed, while in the dynamic case TVBDs are mobile (i.e., personal or portable unlicensed devices). For comparison purposes and to be conform with FCC specifications [12], [13], we assume that fixed and portable unlicensed devices have the same transmission power, i.e., 20 dBm, and share the same TV spectrum bands (i.e., the set of TV channels is $\mathcal{M} = \{21, \ldots, 51\} \setminus \{37\}$), unless otherwise stated. Figure 3 illustrates an example of the TV spectrum that we consider in our numerical analysis. We further assume a free-space path loss model between any two unlicensed devices. Default values of key simulation parameters are shown in Table II.

TABLE II: Default values of key simulation parameters

$\|\mathcal{N}\|$	$\in [1, 20]$
Trans. Power [12], [13]	100 mWatt (20 dBm)
B [12], [13]	6 MHz
r_j	10 Mbps
α_j, β_j, γ_j	1, 1, 0
δ_{ij}	100
d_i	$\in [20, 30]$ Mbps
d_{max}, n_{max}	10, 3
λ [23]	20%

All the results reported hereafter are the Nash equilibria and optimal solutions of the considered scenarios obtained, respectively, by implementing our TVWS spectrum management algorithms in Matlab and OPL (Optimization Programming Language), and solving optimization problems with CPLEX [26].

B. Performance Evaluation

In the following, we measure the effect of the number of TVBDs, the number of available idle TV channels, rate demands as well as the d_{max} and λ parameters on the performance of games $\mathcal{G}_1$ and $\mathcal{G}_2$. We first discuss the results obtained in the static scenario and then those of the dynamic scenario, where TVBDs are personal or portable. Finally, we characterize the efficiency of the equilibria reached by our proposed algorithms by measuring the Price of Anarchy.

Acronym	Description
SBR-DSM	Sequential Best Response-based DSM
BR-DSM	Best Response-based DSM
K-DSM	Krasnoselskij-based DSM
IM-DSM	Imitation-based DSM

TABLE III: Summary of the Distributed Spectrum Management (DSM) algorithms considered in our study.

1) ***Static TVWS scenario:*** In the *static* TVWS scenario, we fix the transmission power to 20 dBm. Parameters α_j, β_j, γ_j, and δ_{ij} are set to 1, 1, 0, and 100, respectively, for all $i \in \mathcal{N}$ and channels $j \in \mathcal{M}$, $d_{max} = 10$, $n_{max} = 3$ and $\lambda = 0.2$ (i.e., 20% of players change strategy in each iteration of the Krasnoselskij-based DSM algorithm, K-DSM).

Note that the SBR-DSM algorithm (sequential best response dynamics) is guaranteed to converge in both games $\mathcal{G}_1$ and $\mathcal{G}_2$ in a finite number of iterations, due to the fact that we demonstrated that these games are potential. In practice we observed that, in all the scenarios we simulated and for all parameters settings, the distributed algorithms we considered in this paper always converge in few iterations to equilibrium conditions (whose quality we will assess hereafter using the Price of Anarchy index). More specifically we observed that, in the worst case, up to 5 iterations are needed for a TVBD to converge to a stable point, while in average less than 3 iterations are sufficient.

We vary the number of TVBDs in the range $[1, 20]$ to show the impact of this parameter on the interference among the devices. We assume that TVBDs rate demands d_i are homogeneous (either equal to 20 or 30 Mb/s), and we consider two cases for the set of idle channels: case (i) $\mathcal{M}$ consists of all idle channels of the spectrum depicted in Figure 3, while in case (ii) $\mathcal{M} = \{21, 22, 28, 29, 30, 35, 36, 38, 39\}$. The aim behind considering case (ii) is to study the system's behavior when a smaller number of idle channels is available for TVBDs, thus increasing the interference.

Figure 4(a) shows the average value of the objective function obtained by the proposed algorithms (SBR-DSM, BR-DSM, K-DSM and IM-DSM, as summarized in Table III) in the context of games $\mathcal{G}_1$ (solid lines in the figure) and $\mathcal{G}_2$ (dotted lines) as a function of the total number of players (unlicensed devices), for d_i equal to 20 Mb/s, considering the entire set of idle channels (case (i)). Similarly, Figure 4(b) shows the same performance measure when a subset of idle channels is available (case (ii)).

Several key findings can be drawn from the observation of these results, namely in terms of the impact of the number of TVBDs, idle channels and rate demands, which we discuss in the following.

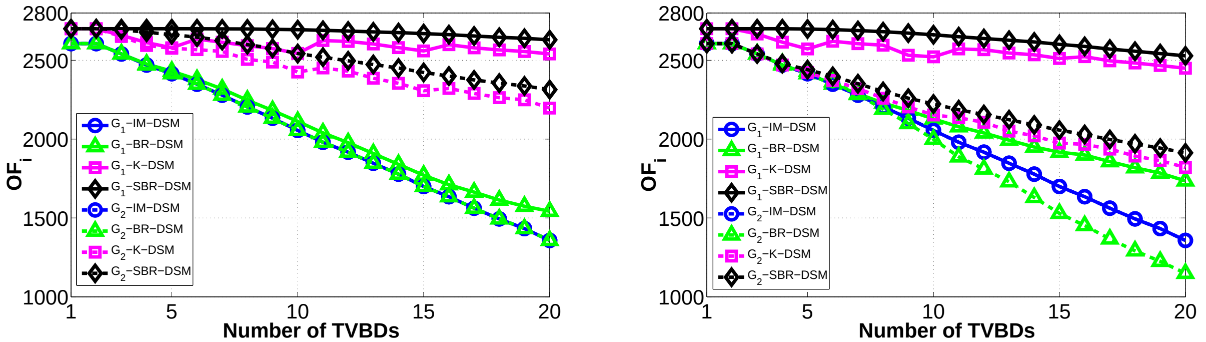
a) Effect of the number of TVBDs: As expected, it can be seen in Figures 4(a) and 4(b) that the objective function decreases when increasing the number of players, and this is in fact due to the increase in the interference between TVBDs. It can also be observed that SBR-DSM and K-DSM have very similar trends for both games, and they exhibit better performance values than IM-DSM and BR-DSM, especially for a number of players higher than 5. SBR-DSM shows the best performance among all the distributed algorithms. In fact, in the sequential version of the Best Response, at each iteration of the algorithm, each device chooses the best channels knowing those chosen by the previous players in the same round or iteration. Therefore, since it relies on a most up-to-date information, it is not surprising that the sequential BR algorithm exhibits the best performance.

b) Effect of the number of idle TV channels: We now investigate the impact of the number of available idle channels on the performance of the proposed games and distributed algorithms. We observe from Figure 4(a) and Figure 4(b) that a player gets, on average, in case (ii) an objective function value lower than that perceived in case (i). For example, under SBR-DSM and K-DSM, when game $\mathcal{G}_2$ is played by 20 TVBDs, each TVBD achieves an objective function value under case (i) that is, on average, 1.3 times higher than the one obtained under case (ii). This is due to the fact that in case (i) the set of strategies is larger than the one in case (ii) and hence TVBDs can better optimize their performance in the former case. This particularly applies to SBR-DSM and K-DSM. However, IM-DSM exhibits the same trend in the two cases, and this can be explained by the fact that the imitation algorithm is more sensible to the number of players than the size of the set of strategies that players can explore.

c) Effect of traffic distributions and rate demands: We measure the impact of the traffic pattern by considering different realistic distributions for the traffic demand, as done in [16]. More specifically, we consider: a) deterministic traffic with rate of 20 Mb/s, b) uniformly distributed traffic with rate between 10 and 30 Mb/s, and c) truncated normally distributed traffic with mean and standard deviation of 20 and 5 Mb/s, respectively. The average value of the objective function versus the number of TVBDs for uniformly distributed traffic[2] is reported in Figure 5 for all the algorithms presented in the paper. By comparing Figures 4 and 5, it can be observed that the impact of different distributions is limited (i.e., practically almost negligible) in case (i), where all channels are used (Figure 4(a) and 5(a)). This impact becomes slightly more noticeable in case (ii), where fewer channels are considered (Figure 4(b) and 5(b)) and the number of TVBDs competing to access these channels is large. Moreover, the number of iterations required to reach a Nash Equilibrium (NE) in the deterministic traffic case as well as when the traffic pattern varies according to a normal/uniform distribution are very similar, i.e., less than 3 iterations, on average, and up to 5 in the worst case. Therefore, we can conclude that our proposed algorithms are quite robust against different traffic patterns.

We further evaluate the effect of the rate demand on the proposed algorithms, considering two values of d_i (20 Mb/s and 30 Mb/s). Since the maximum number of channels that can be chosen by a player, n_{max}, is fixed to 3, we observe that the impact of d_i is quite limited, and this is especially true when a small number of players is involved. However, the impact of d_i is greater with larger values of n_{max}.

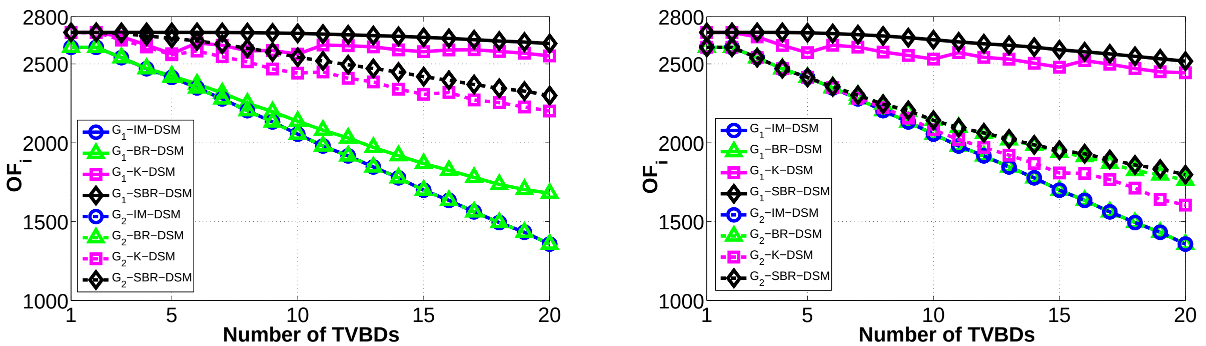
[2]Note that we obtain similar results when we assume that the traffic follows a truncated normal distribution.



(a) Deterministic traffic, d_{max}=10, case (i)

(b) Deterministic traffic, d_{max}=10, case (ii)

Fig. 4: Static TVWS scenario: Average objective function values (the players' total utility) as a function of the number of TVBDs ($[1, 20]$), and for two different sets of available idle channels (traffic demand $d_i = 20$ Mb/s).



(a) Uniform dist., d_{max}=10, case (i)

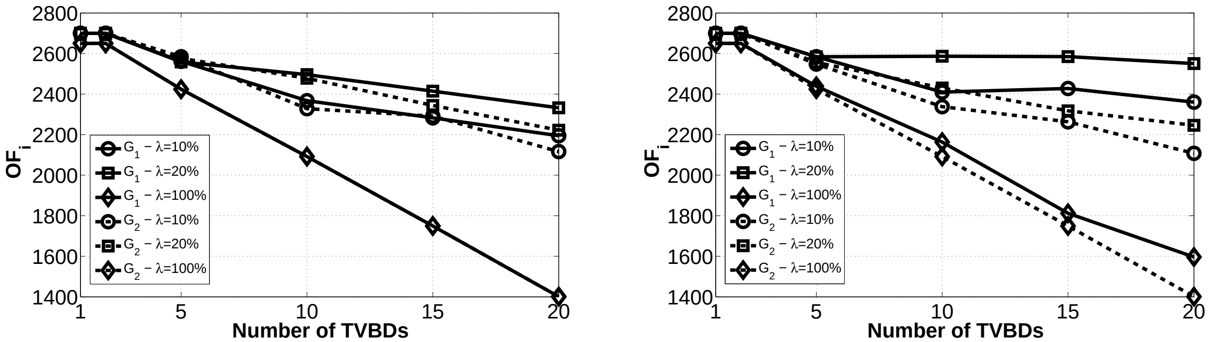
(b) Uniform dist., d_{max}=10, case (ii)

Fig. 5: Static TVWS scenario: Average objective function values (the players' total utility) versus the number of TVBDs ($[1, 20]$), and for two different sets of available idle channels (uniformly distributed traffic with rate between 10 and 30 Mb/s).

d) Effect of parameters d_{max} and λ : Finally, we evaluate the effect of parameters λ and d_{max} on the performance of our proposed distributed algorithms. Recall that λ represents the percentage of players that change strategy in each iteration of the K-DSM algorithm, and d_{max} is the maximum allowed frequency span between the channels, specifically, between the first and the last channel chosen by a TVBD. Therefore, we consider a TVWS scenario varying the number of TVBDs in the range $[1, 20]$, fixing the rate demand to 20 Mb/s and considering different values of λ as well as of d_{max}.

Figures 6(a) and 6(b) illustrate the average value of the objective function obtained by K-DSM in the channel aggregation as well as in the channel bonding game as a function of the number of TVBDs, considering the set of idle channels $\mathcal{M}$ depicted in Figure 3, for three different values of λ: 10%, 20% and 100% (this latter corresponds to the Best Response algorithm, BR-DSM) and d_{max} equal to 5 and 10, respectively. It can be observed that K-DSM exhibits better performance than BR-DSM in both games when λ is equal to 10% and 20%. The same behavior is observed with the two proposed games. In fact, when a small fraction of players (10-20%) change their strategy (the upper curves), the achieved equilibria are significantly more efficient than those obtained when all players adapt their choices simultaneously (the lower curves in both Figures 6, i.e. $\lambda = 100\%$). The gap between K-DSM and BR-DSM increases when increasing the number of players. For instance, when this latter is equal to 20 and $d_{max} = 5$, the percentage gap between K-DSM and BR-DSM is approximately 71% for $\lambda = 20\%$ and 57% for $\lambda = 10\%$. Furthermore, the channel aggregation game (bold lines in Figures 6) outperforms the channel bonding game (dashed lines), and the gap between the average objective function values is more evident when the maximum frequency span doubles (Figure 6(b)). On the other hand, the performance of the channel aggregation game improves when increasing d_{max}. This can be explained by the fact that each TVBD has a larger spectrum to explore when $d_{max} = 10$ and hence can likely better optimize its objective function.

Finally, Figure 7 shows the impact of the parameter d_{max} on the objective function perceived by players of the channel aggregation game for two values of the maximum frequency span (viz., 5 and 10). It can be seen that, under all the proposed algorithms, with $d_{max} = 10$ the players obtain a utility which is higher than the one obtained with $d_{max} = 5$. As argued previously, when the frequency spectrum is large, the players succeed in ameliorating their outcomes at the cost of a larger space to explore. In particular, for K-DSM, the percentage gap in the outcome is equal to 15% when the number of devices is equal to 20.



(a) $d_{max} = 5$

(b) $d_{max} = 10$

Fig. 6: Average objective function values as a function of the number of TVBDs ($[1, 20]$), available idle channels ($\{21, \ldots, 51\} \setminus \{37\}$) and for three different λ values, viz., 10%, 20% and 100% (this latter corresponds to the Best Response algorithm, BR-DSM).

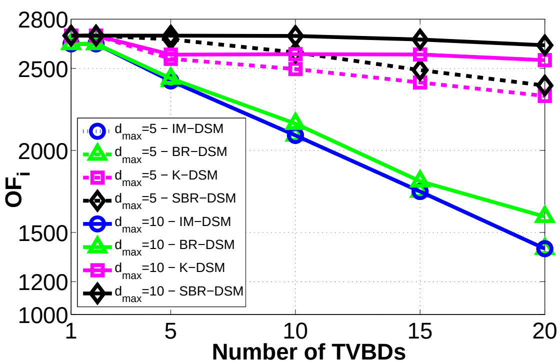


Fig. 7: Channel Aggregation Game - Static TVWS scenario: Average objective function values as a function of the number of TVBDs ($[1, 20]$), and for two different maximum frequency spans (viz., 5 and 10) and λ=20%.

*2) **Dynamic TVWS scenario**:* In the *dynamic* TVWS scenario, we simulate TVBDs' mobility using the random way-point model [27], [28], as commonly assumed in the literature. In particular, we divide the operating time of the system into 10 consecutive time epochs, and for each epoch we compute the random displacements of all mobile devices according to a displacement vector. The mobile device speed is set to 1 m/s, while the time epoch duration is fixed to 10 s. Indeed, mobility can generate in each epoch some modifications in the interference matrix among devices, and as a consequence TVBDs can change strategies from one time epoch to another, if necessary, to improve their expected utility values.

Figure 8 illustrates an example of a dynamic TVWS scenario where the positions of 20 mobile TVBDs are generated, according to the random way-point model, on a square area of 500×500 m^2 at time epochs 1, 5 and 10. For the sake of clarity, in this figure we show only 20 devices and 3 out of 10 time epochs.

Figures 9(a) and 9(b) show the average value of the objective function of all devices, considering, respectively, two different sets of idle channels (case (i) and case (ii)) and fixing the rate demand to 20 Mb/s. In this dynamic scenario, the plotted values are obtained by averaging over all time epochs. The results measured in the dynamic TVWS scenario confirm the trends observed in the static scenario. Specifically, the interference can be reduced by increasing the number of idle channels and decreasing the number of TVBDs (or equivalently, by increasing the spatial reuse). Finally, we can observe that in the considered scenarios the density of TVBDs affects the network performance more than mobility, since all proposed algorithms allocate idle TV channels on per-time-epoch basis, taking into account the interference matrix and the congestion measured in each epoch.

*3) **Efficiency of the Nash equilibria - Price of Anarchy**:* We now study the efficiency of the Nash equilibria reached in our proposed games by comparing them to the socially optimal solutions, through the determination of the PoA.

Socially optimal solutions maximize the sum of all TVBDs' objective functions, i.e., they maximize $\sum_{i \in \mathcal{N}} \{OF_i = U_i - J_i\}$, subject to constraints (5)-(8) for game $\mathcal{G}_1$ and constraints (5)-(9) for $\mathcal{G}_2$, $\forall i \in \mathcal{N}$, where J_i and U_i are given in (1) and (3), respectively. The PoA is defined as the ratio between the objective function value of this solution and that of the worst NE.

We determine hereafter the PoA for static and dynamic TVWS scenarios. The parameters settings are the same as described in the previous sections.

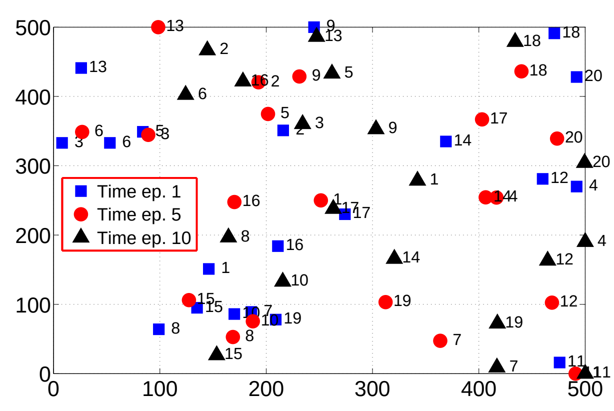
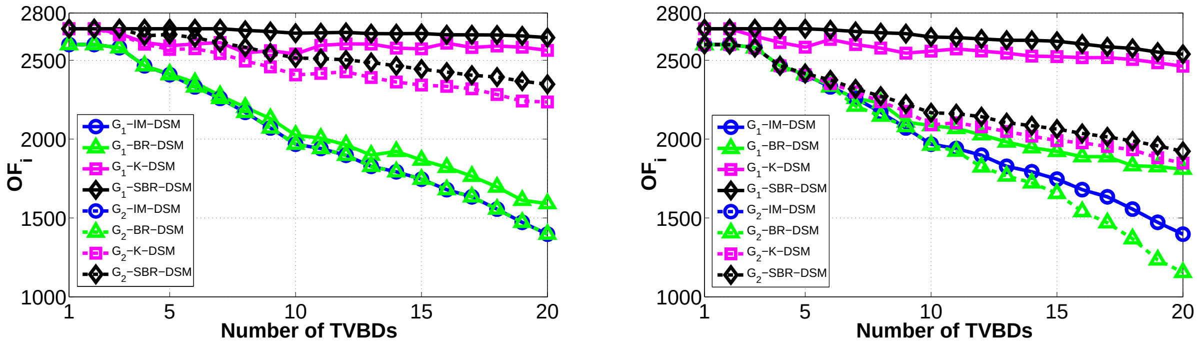
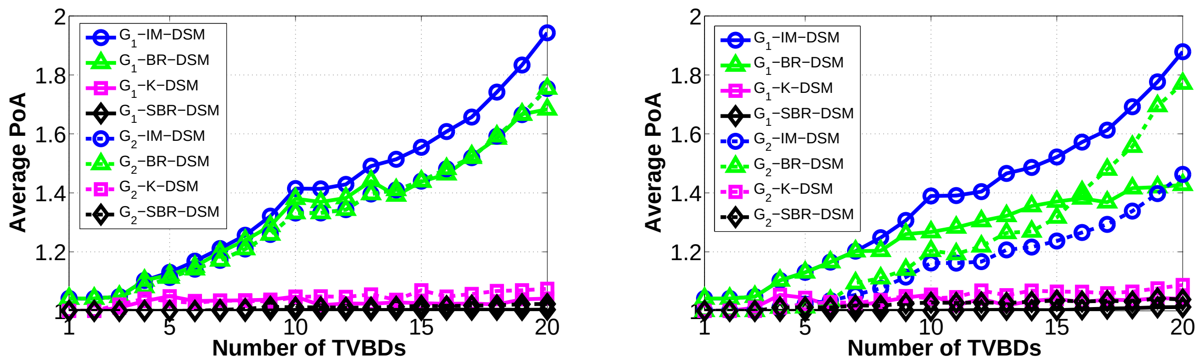


Fig. 8: An example of a dynamic TVWS scenario where the positions of 20 mobile TVBDs are generated on a square area of 500×500 m^2 for time epochs 1, 5 and 10.



(a) d_{max}=10, case (i) (b) d_{max}=10, case (ii)

Fig. 9: Dynamic TVWS scenario: Average objective function values as a function of the number of TVBDs ($[1, 20]$), and for two different sets of available idle channels.



(a) d_{max}=10, case (i) (b) d_{max}=10, case (ii)

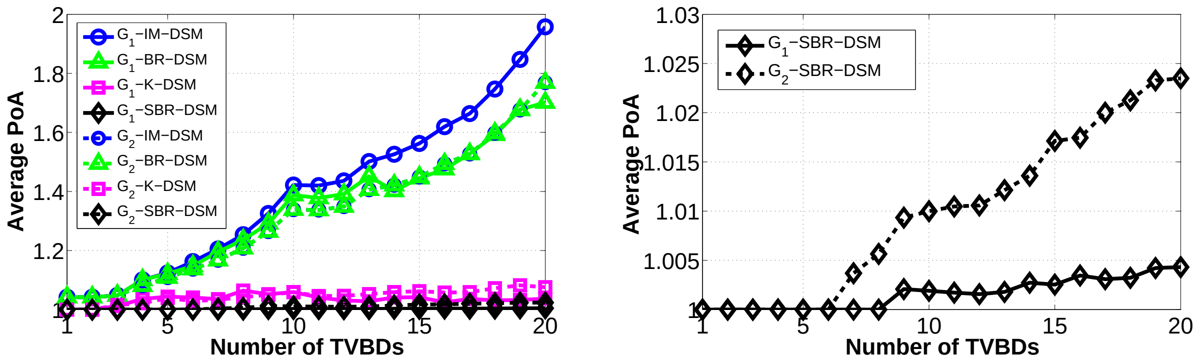
Fig. 10: Static TVWS scenario: Average PoA values as a function of the number of TVBDs ($[1, 20]$), and for two different sets of available idle channels (traffic demand $d_i = 20$ Mb/s).

Figure 10(a) shows the average value of the PoA for *static* TVWS scenarios measured under the Best Response, the Krasnoselskij, and the Imitation-based algorithms implementing the two proposed games, while varying the number of fixed devices in the range $[1, 20]$ and considering case (i). Similarly, Figure 10(b) shows the same performance metric measured under case (ii). As expected, the PoA increases with the number of TVBDs. Under SBR-DSM and K-DSM, the PoA is very small and remains close to 1. On the other hand, with the Imitation and the BR algorithms, the PoA is small when the number of devices is small (less than 5 devices) and then takes larger values (by increasing quasi-linearly) when the number of TVBDs increases.

Conversely, Figure 11(a) shows the average value of the PoA obtained with both games $\mathcal{G}_1$ and $\mathcal{G}_2$ in the *dynamic* TVWS scenarios as a function of the number of TVBDs, considering all idle channels (case (i)) and fixing the rate demand to 20 Mb/s. Figure 11(b) shows the PoA value focusing only on the Sequential Best Response algorithm. It can be clearly seen that the PoA slightly increases when increasing the number of TVBDs, while remaining very small (below 1.025 in the case of 20 players). These results are completely in line with those observed in the static scenarios. As argued previously, the impact of the density of TVBDs is stronger than that of mobility, and therefore the PoA obtained in the dynamic scenarios confirms our findings for the static ones. Finally, in all the considered scenarios, and for all parameters' settings, the PoA remains low (it is, in fact, always lower than 2). This trend is indeed due to the good properties provided by the proposed TVBD's objective function and confirms that our distributed, game theoretic approaches can achieve good results that are close to the optimum.

Remarks:

- The proposed algorithms do not involve complicated operations and their iteration duration (i.e., the rate at which these algorithms are triggered) can be configured (i.e., decreased) to reduce energy consumption. Their signalling overhead is also quite low and the number of iterations is small (less than 3 iterations, on average). Thus, the signalling overhead and time delay remain significantly small compared with centralized or global channel allocation approaches.
- To accommodate traffic changes, one solution would be to assume that TVBD traffic varies according to a known probability distribution (as often done in the literature). Then, we can discretize this information on a finite number of scenarios and divide the operation time of the system into a number of sequential time slots. After that, we can develop some (distributed) stochastic optimization approaches to address this channel allocation problem



(a) d_{max}=10, case (i)

(b) SBR-DSM; d_{max}=10, case (i)

Fig. 11: Dynamic TVWS scenario: Average PoA values as a function of the number of TVBDs ($[1, 20]$), and for two different sets of available idle channels.

under traffic uncertainty. At this point, and at each time slot, we can execute our proposed algorithms, which exhibit a very short convergence time (few iterations on average, as discussed before), and allocate in a fast manner the necessary number of channels to TVBDs to accommodate traffic changes.

VI. Conclusion

In this paper we addressed the TV spectrum management problem considering non cooperative games among TV bands devices (fixed and portable), taking into accurate account users mobility and spatial reuse, along with Adjacent-Channel Interference between different devices' transmissions. We considered both practical cases where contiguous or non-contiguous channels can be accessed by TVBDs, imposing realistic constraints like a maximum frequency span between the chosen channels. To obtain efficient Nash equilibrium solutions, we introduced a congestion cost function that aims at reducing interference between unlicensed devices. We demonstrated under specific conditions on cost function parameters that the guard band-aware TVWS management game admits a potential function, and therefore we used a Best Response algorithm to converge fast to Nash equilibrium points. Furthermore, we proposed an effective algorithm based on Imitation dynamics, where a TVBD probabilistically imitates successful selection strategies of other TVBDs, in order to improve his objective function.

We evaluated the performance of the proposed games and algorithms considering both static and dynamic TVWS scenarios (characterized by users' mobility), illustrating their sensitivity to different key parameters, including the number of TVBDs, the number of available idle blocks, and the rate demands, among others. Numerical results showed that the proposed game theoretical approaches and algorithms perform very well when the available TV resources are limited and the number of TVBDs is high. In fact, they well approach the performance obtained by a centralized scheme, where the Database Operator collects TVBDs demands and allocates the spectrum so as to maximize the social welfare (the total utility experienced by all users). For these reasons, our game theoretical framework provides a very effective tradeoff between efficient TV spectrum use and reduction of interference between TVBDs.

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Appendix A
Potential Function and Existence of NE

We now demonstrate that under some conditions to be specified later, games $\mathcal{G}_1$ and $\mathcal{G}_2$ admit a potential function Φ. Indeed, if a potential function exists, these games are *potential* [21], and possess at least one pure-strategy Nash Equilibrium (NE). Hence, a Best Response algorithm can be used to converge to a NE.

Proposition A.1. *Games $\mathcal{G}_1$ and $\mathcal{G}_2$ admit a potential function Φ, which is given by the following expression:*

$$\Phi = \sum_{i \in \mathcal{N}} [U_i - \frac{1}{2} J_i] - \frac{1}{2} \sum_{i \in \mathcal{N}} \sum_{j \in \mathcal{M}} [\alpha_j r_j^2 x_{ij}^2 + r_j \gamma_j x_{ij}]. \tag{12}$$

Proof: Assume that $\beta_j = 1$ and $e_{ki}^{(j)} = e_{ik}^{(j)}\ \forall i, k \in \mathcal{N}, j \in \mathcal{M}$, i.e., the interference between TVBDs is symmetric (a natural assumption). Function Φ is a *potential function* if it satisfies the following condition for each player i, each multi-strategy $s = \{s_1, \ldots, s_i, \ldots, s_{|\mathcal{N}|}\} = \{s_i\} \cup \{s_{-i}\}$, and each strategy $v_i \neq s_i$:

$$\Phi(s_i, s_{-i}) - \Phi(v_i, s_{-i}) = OF_i(s_i, s_{-i}) - OF_i(v_i, s_{-i}). \tag{13}$$

Let Φ_j and $OF_{i,j}$ be the potential function and TVBD i objective function, respectively, for channel j. Hence, $\Phi = \sum_{j \in \mathcal{M}} \Phi_j$ and $OF_i = \sum_{j \in \mathcal{M}} OF_{i,j}$.

Consider Φ_j:

$$\begin{aligned}
&\Phi_j(x_{ij}, x_{-ij}) \\
&= \sum_{k \in \mathcal{N}: k \neq i} U_{k,j}(x_{kj}) + U_{i,j}(x_{ij}) \\
&\quad - \frac{1}{2} \sum_{k \in \mathcal{N}} r_j x_{kj} \cdot [\alpha_j \cdot (\sum_{l \in \mathcal{N}} r_j e_{lk}^{(j)} x_{lj}) + \gamma_j] \\
&\quad - \frac{1}{2} \sum_{k \in \mathcal{N}} [\alpha_j (r_j x_{kj})^2 + r_j \gamma_j x_{kj}] \\
&= \sum_{k \in \mathcal{N}: k \neq i} U_{k,j}(x_{kj}) + U_{i,j}(x_{ij}) \\
&\quad - \frac{\alpha_j r_j^2}{2} \Big\{ \sum_{k,l \in \mathcal{N}: k,l \neq i} e_{lk}^{(j)} x_{lj} x_{kj} \\
&\quad + \sum_{k \in \mathcal{N}: k \neq i} e_{ik}^{(j)} x_{ij} x_{kj} + \sum_{l \in \mathcal{N}: l \neq i} e_{li}^{(j)} x_{lj} x_{ij} \\
&\quad + \sum_{k \in \mathcal{N}: k \neq i} 2x_{kj}^2 + 2x_{ij}^2 \Big\} \\
&\quad - \sum_{k \in \mathcal{N}: k \neq i} r_j \gamma_j x_{kj} - r_j \gamma_j x_{ij}.
\end{aligned} \tag{14}$$

Since the interference between TVBDs is symmetric ($e_{ki}^{(j)} = e_{ik}^{(j)}$, $\forall i, k \in \mathcal{N}, j \in \mathcal{M}$), we can further simplify the expression of Φ_j:

$$\begin{aligned}
&\Phi_j(x_{ij}, x_{-ij}) \\
&= \sum_{k \in \mathcal{N}: k \neq i} U_{k,j}(x_{kj}) + U_{i,j}(x_{ij}) \\
&\quad - \frac{\alpha_j r_j^2}{2} \Big\{ \sum_{k,l \in \mathcal{N}: k,l \neq i} e_{lk}^{(j)} x_{lj} x_{kj} \\
&\quad + 2 \sum_{k \in \mathcal{N}: k \neq i} e_{ki}^{(j)} x_{ij} x_{kj} + \sum_{k \in \mathcal{N}: k \neq i} 2x_{kj}^2 + 2x_{ij}^2 \Big\} \\
&\quad - \sum_{k \in \mathcal{N}: k \neq i} r_j \gamma_j x_{kj} - r_j \gamma_j x_{ij}.
\end{aligned} \tag{15}$$

It is easy to verify that any unilateral deviation of TVBD i on a channel j, which gives rise to a change in its objective function, is exactly reflected in function Φ_j. In fact, the following equality holds:

$$\begin{aligned}
&\Phi_j(x_{ij}, x_{-ij}) - \Phi_j(y_{ij}, x_{-ij}) \\
&= U_{i,j}(x_{ij}) - U_{i,j}(y_{ij}) \\
&\quad - [\alpha_j r_j^2 (x_{ij}^2 - y_{ij}^2) - r_j \gamma_j (x_{ij} - y_{ij})] \\
&\quad - (\sum_{k \in \mathcal{N}, k \neq i} \alpha_j r_j^2 e_{ki}^{(j)} x_{kj})(x_{ij} - y_{ij}) \\
&= OF_{i,j}(x_{ij}, x_{-ij}) - OF_{i,j}(y_{ij}, x_{-ij}).
\end{aligned} \tag{16}$$

Hence, by summing up over all wireless channels j, we prove that (13) and (14) hold, and that the TVWS spectrum management games admit a potential function Φ. □